The Eagle-1 time synchronization scheme

Conrad Rößler, Bastian Hacker, Kevin Günthner, and Christoph Marquardt

The Eagle-1 QKD Protocol – from science to application Workshop

13.11.2024

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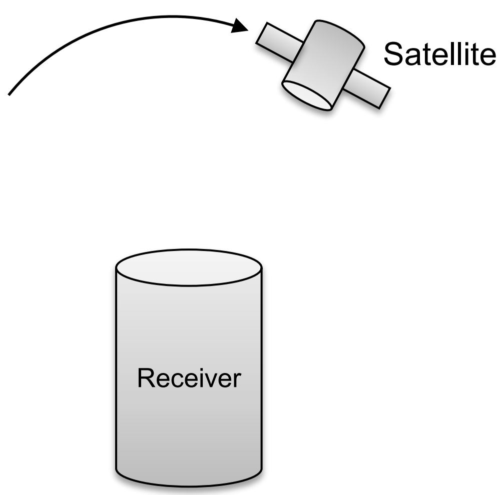
Clock recovery

- Main challenge in satellite-based QKD: frequency drift due to Doppler shift + high loss
- Typical values for LEO satellites:
  - Doppler shift 2e-5, chirp 4e-7 / s
  - 40-60 dB loss



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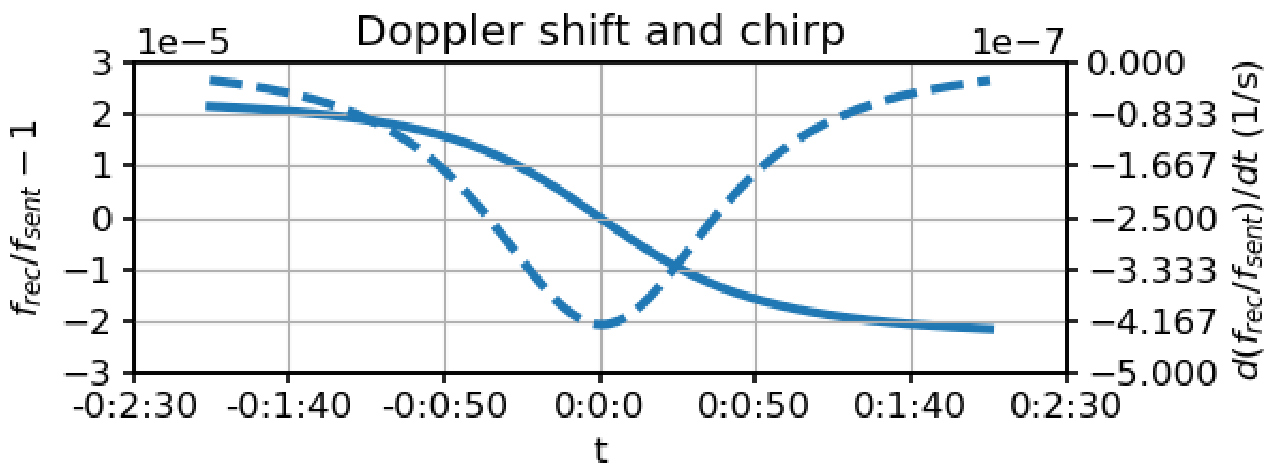

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Clock recovery

- Less interfaces = better
  - Don’t rely on classical communication
  - Do sync in the quantum channel

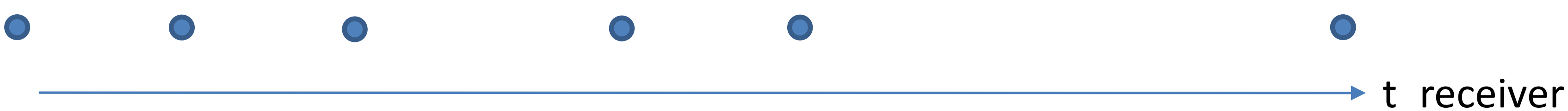
Clock recovery

- Less interfaces = better
  - Don’t rely on classical communication
  - Do sync in the quantum channel
- We need bright pulses for the phase lock anyway [1]
  - → use them for the clock recovery in addition!

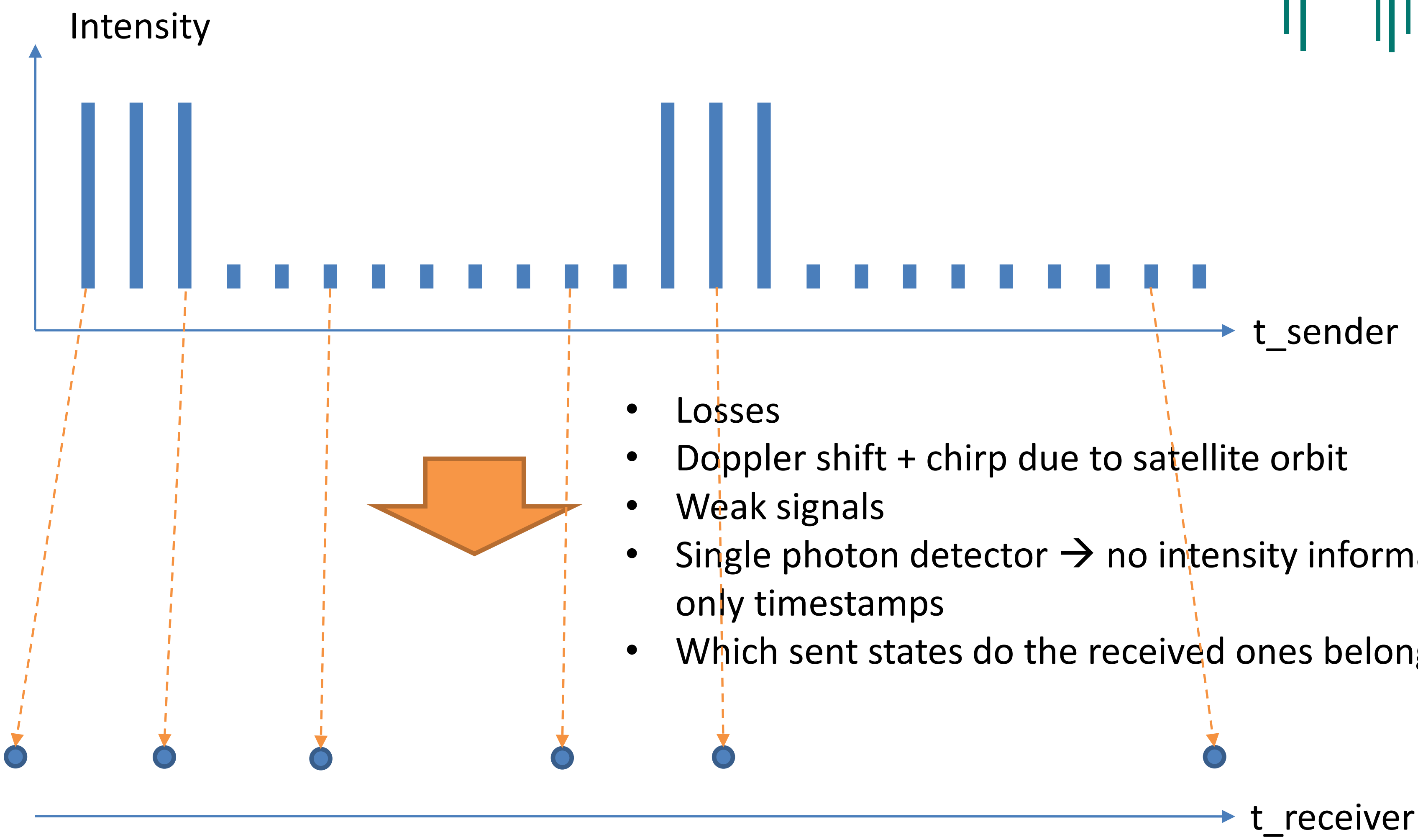
[1] Bastian Hacker et al 2023
New J. Phys. 25 113007

Clock recovery

- Losses
- Doppler shift + chirp due to satellite orbit
- Weak signals
- Single photon detector → no intensity information, only timestamps
- Which sent states do the received ones belong to?



Clock recovery



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Clock recovery

- Measurements and output:

t (ps)	Ch
123	1
45691	3
150816	4
280692	2
…	…

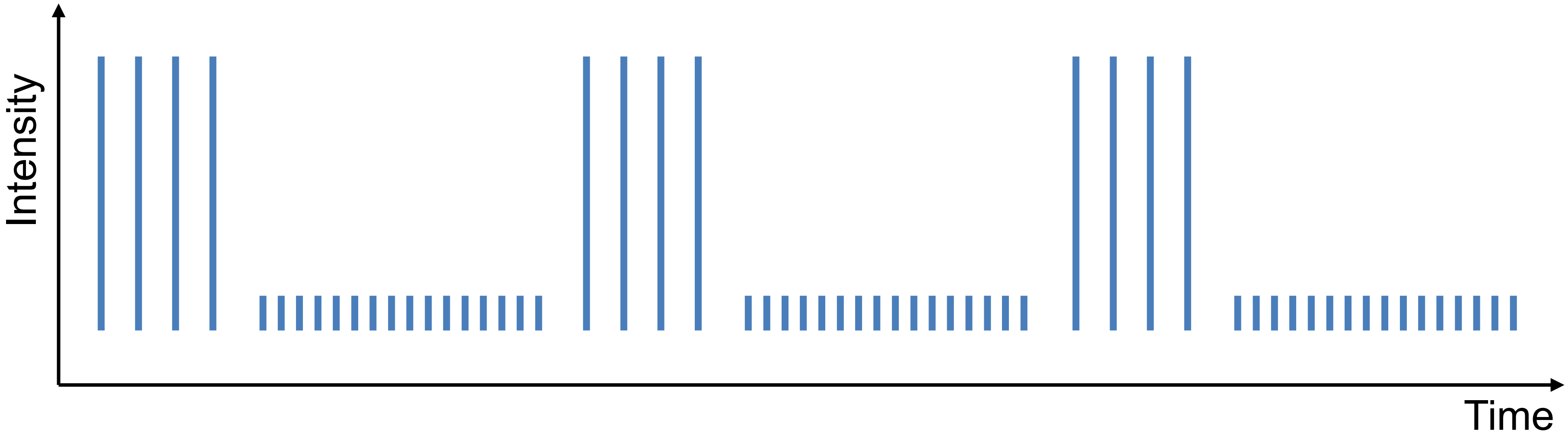
→ Recv SW →

Frame	Index	Type	t in symbol (ps)
1	00016	Ref	40
2	56778	Sig	123
2	97561	Sig	380
4	08912	Ref	160
…	…	…	…

→ UDP → QPS

Clock recovery

- Frame? Index? Type?



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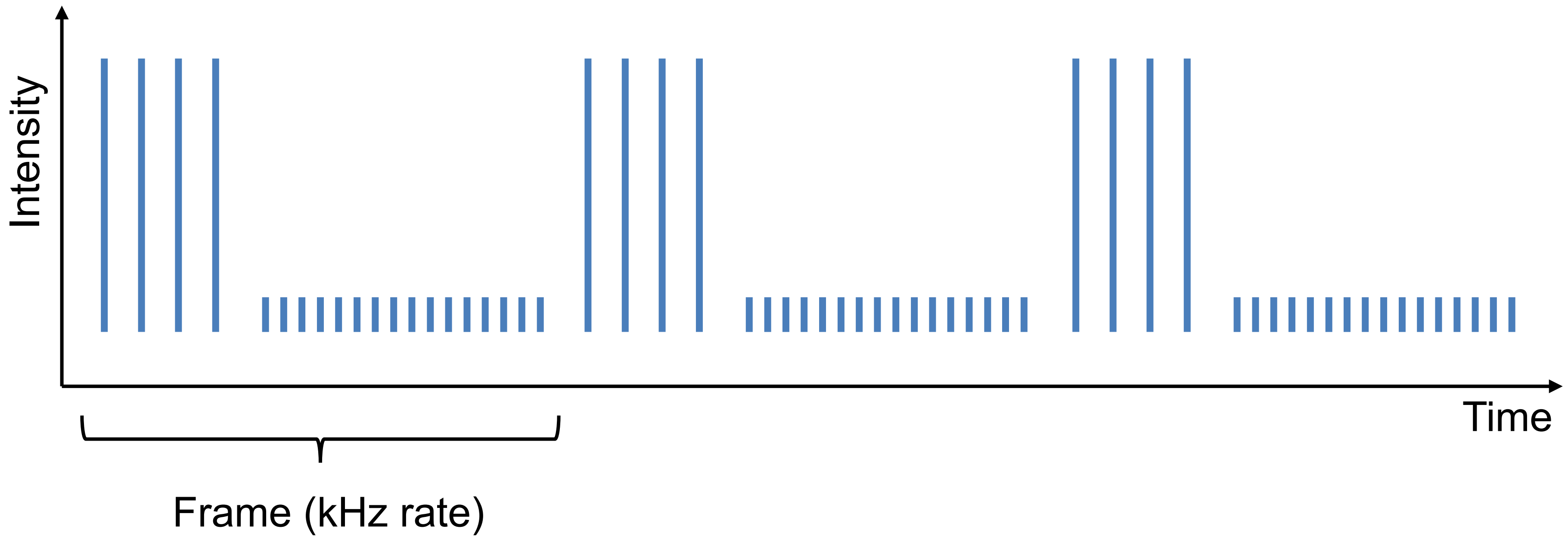

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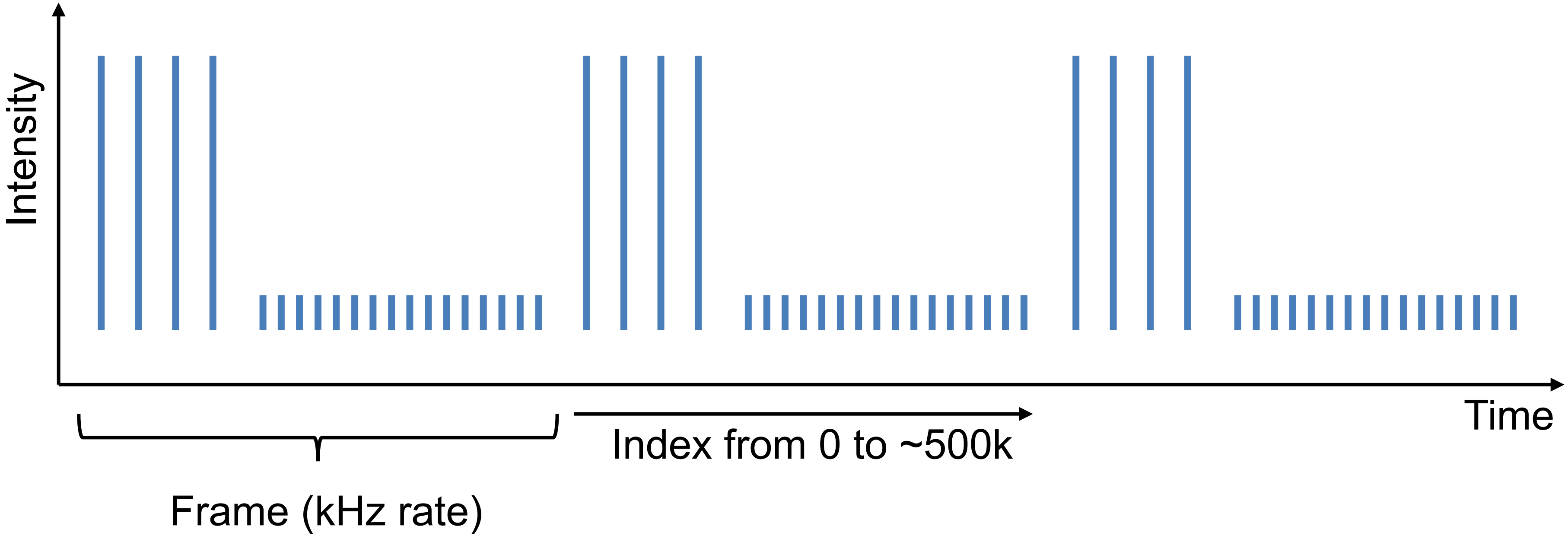

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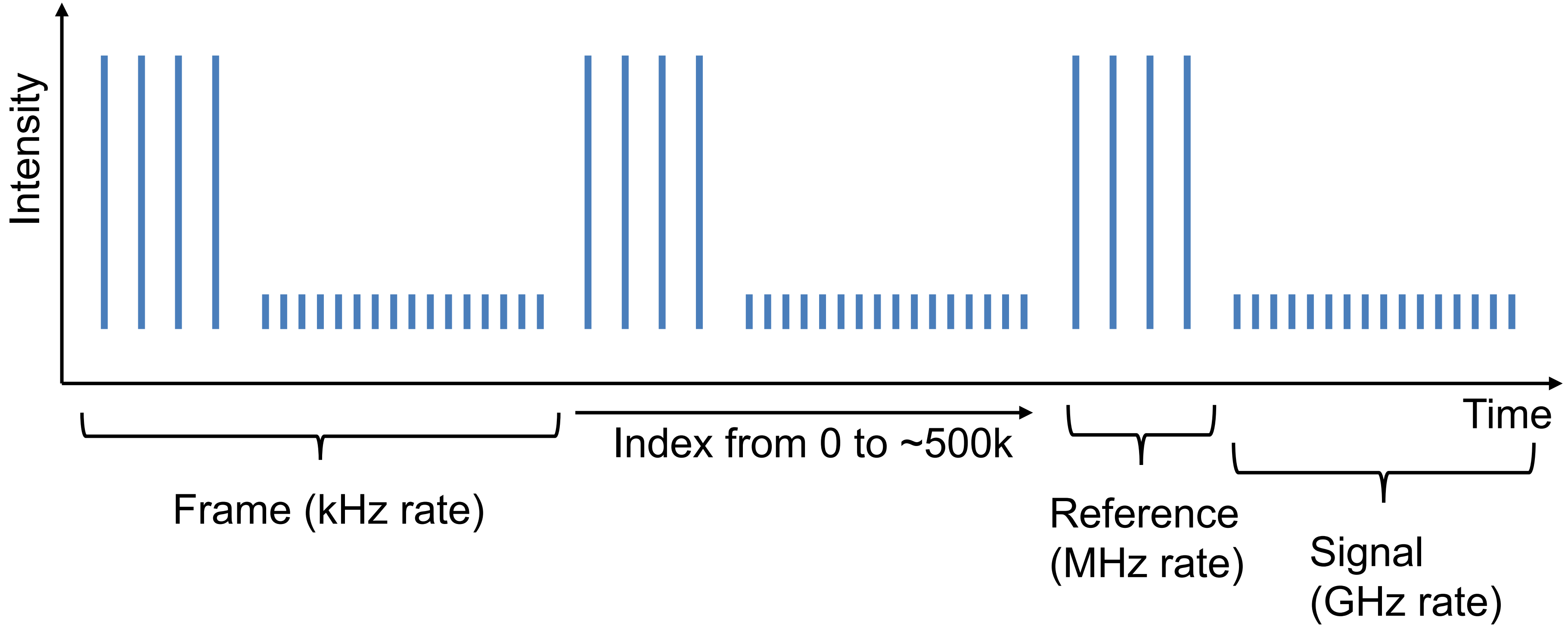

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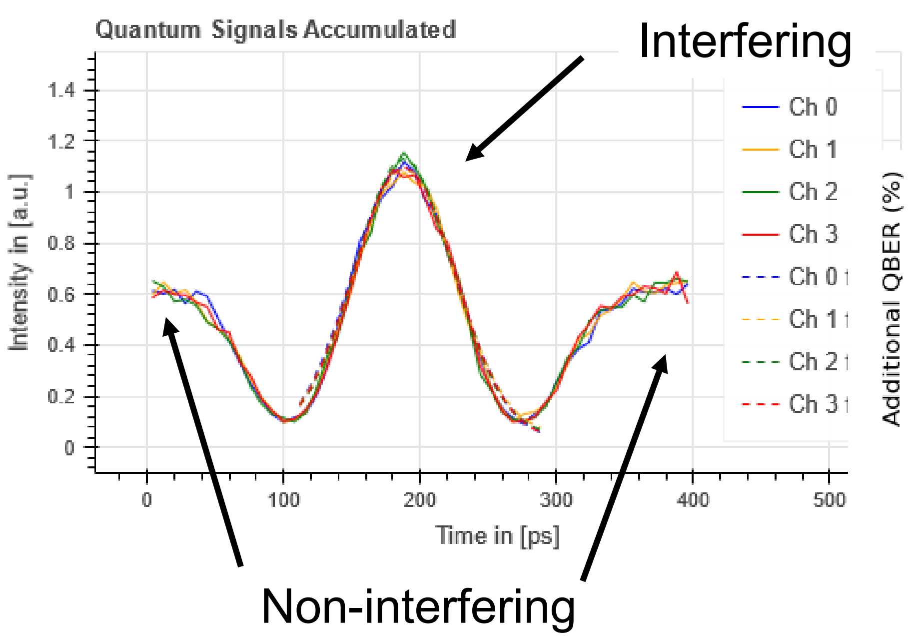
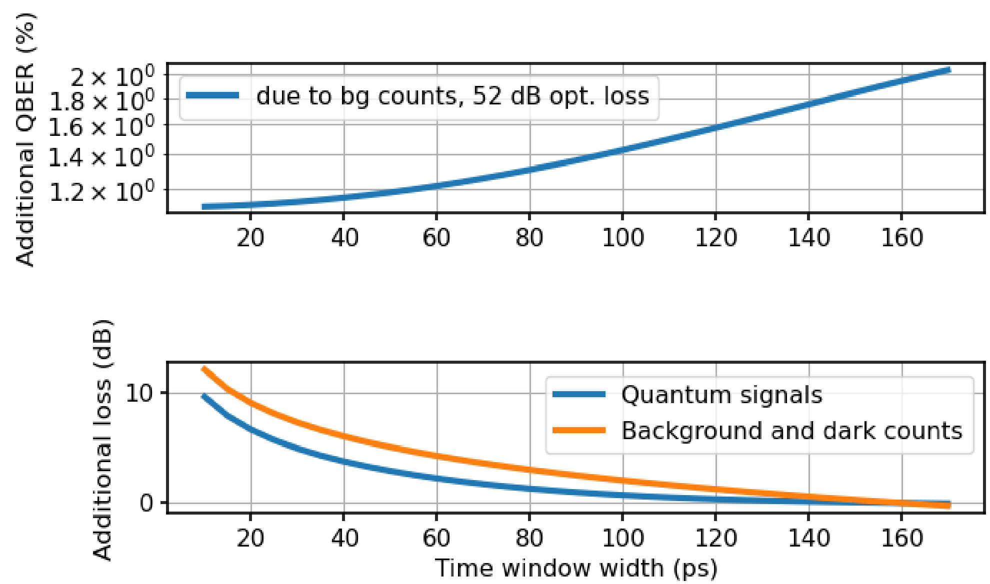
Clock recovery

- Why do we need t within symbol?
- Quantum pulses have different temporal distribution than dark counts → time window





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Clock recovery

- What we want to do:
  - Relative sync: frequencies!
    - Determine exact signal rate
  - Absolute sync: offsets!
    - Frame number? ~200 µs
    - Index of quantum pulse? ~400 ps
    - Where is the “middle” of a quantum pulse? ~10 ps

Relative sync

- How to proceed?
- GHz signal rate
- Timing requirement → < 1 s time to recover clock → work on ~$t_{total} = 100$ ms blocks
- With reference pulses: < 1 kCounts/block

Relative sync

- Required period accuracy ΔT?
- Period T
  - → offset after sifting block $\ll T$
  - → offset after one period: $\ll T \cdot T/t_{total}$
- Per period T: $\Delta T < 1\,fs$
- Translated into frequency:

$$\Delta F = \frac{\Delta T}{T} \cdot F < 10\,Hz$$

Clock recovery

- FFT?
  - t_{total} = 100 ms: ~1e9 entries
  - FFTing a 10^9 array with all 0s except for few k 1s?

Clock recovery

- FFT?
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```
>>> np.fft.fft(np.zeros(int(1e9),dtype=np.uint8))
Traceback (most recent call last):
  File "<stdin>", line 1, in <module>
  File "<__array_function__ internals>", line 200, in fft
  File "C:\ProgramData\anaconda3\Lib\site-packages\numpy\fft\_pocketfft.py", line 215, in fft
    output = _raw_fft(a, n, axis, False, True, inv_norm)
             ^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^

  File "C:\ProgramData\anaconda3\Lib\site-packages\numpy\fft\_pocketfft.py", line 70, in _raw_fft
    r = pfi.execute(a, is_real, is_forward, fct)
        ^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^

MemoryError
```

Relative sync

- FFT?
  - Frequency already known to some degree, no need to check every frequency from 0 to 1/T with Hz resolution!
  - Arrays too long / computation too slow
- → **Epoch folding!**

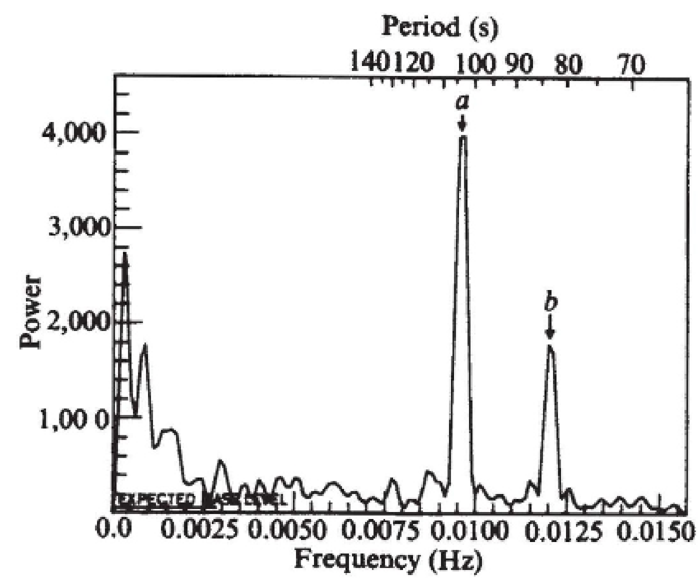
What is epoch folding?

- Rosenberg et al., 1975
- Astrophysics
- Recent renaissance within the QKD context
  - BayernQSat internal documentation, 2021
  - [1] Spiess and Steinlechner, 2023
  - [2] Zahidy et al., 2023

letters to nature

Observations of a transient X-ray source with a period of 104 s

Received June 13; accepted July 18, 1975.



An epoch-folding analysis yields the best estimate for the period of 104.14 ± 0.16 s (1σ). Figure 3 shows the observed

What is epoch folding?

- Expected period T_{guess}:
  - For each timestamp t, get $t \,\%\, T_{guess}$ and sort into histogram
- Repeat for several T_{guess}
- Histogram with highest variance belongs to correct period

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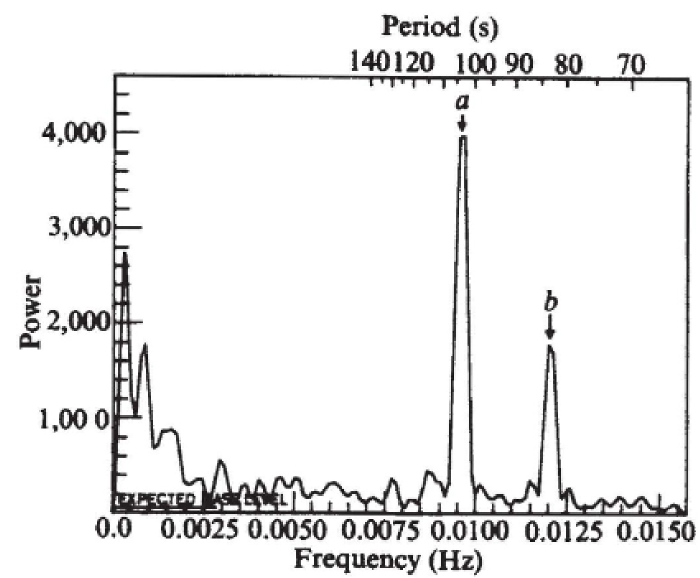

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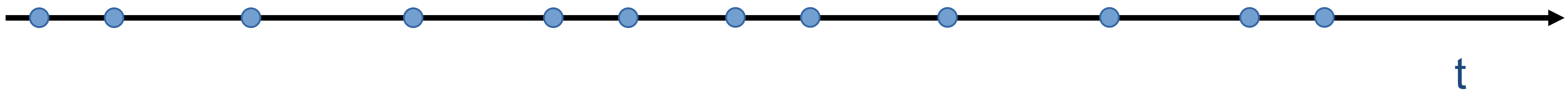

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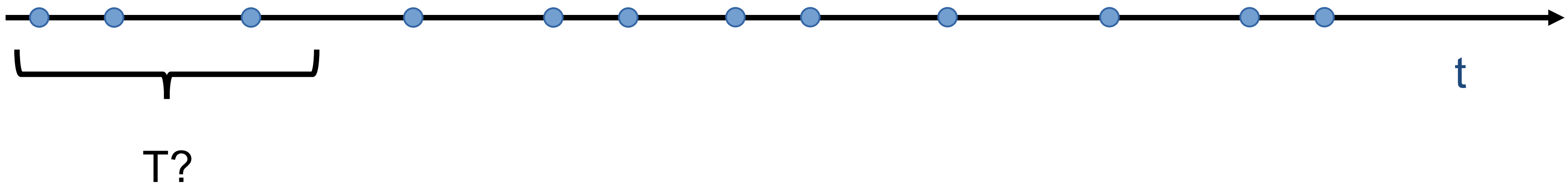

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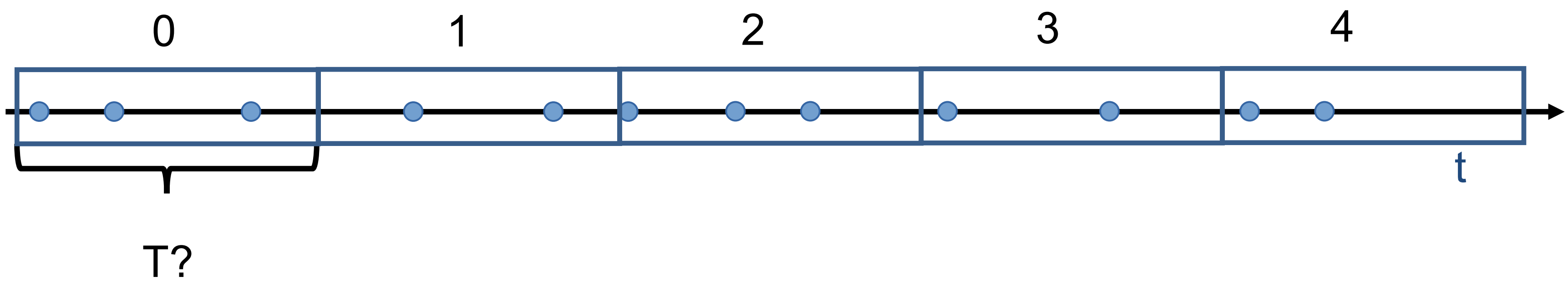

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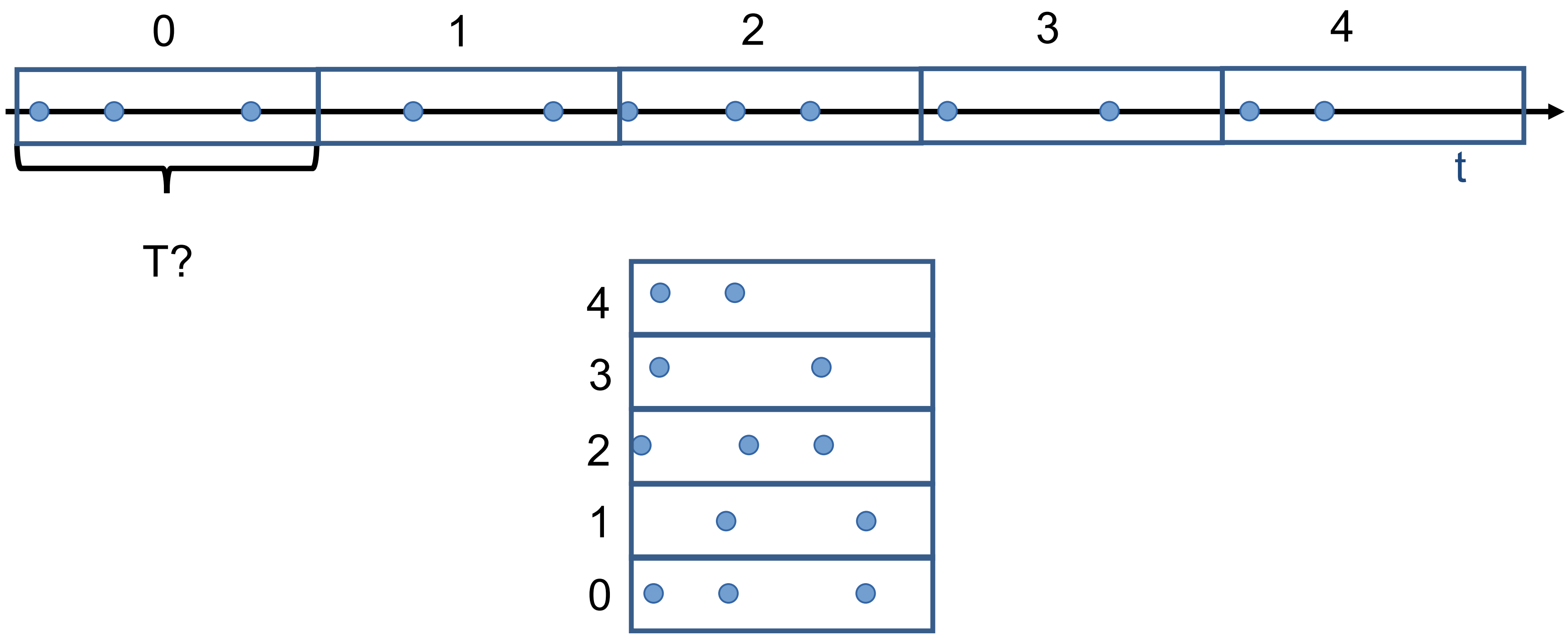

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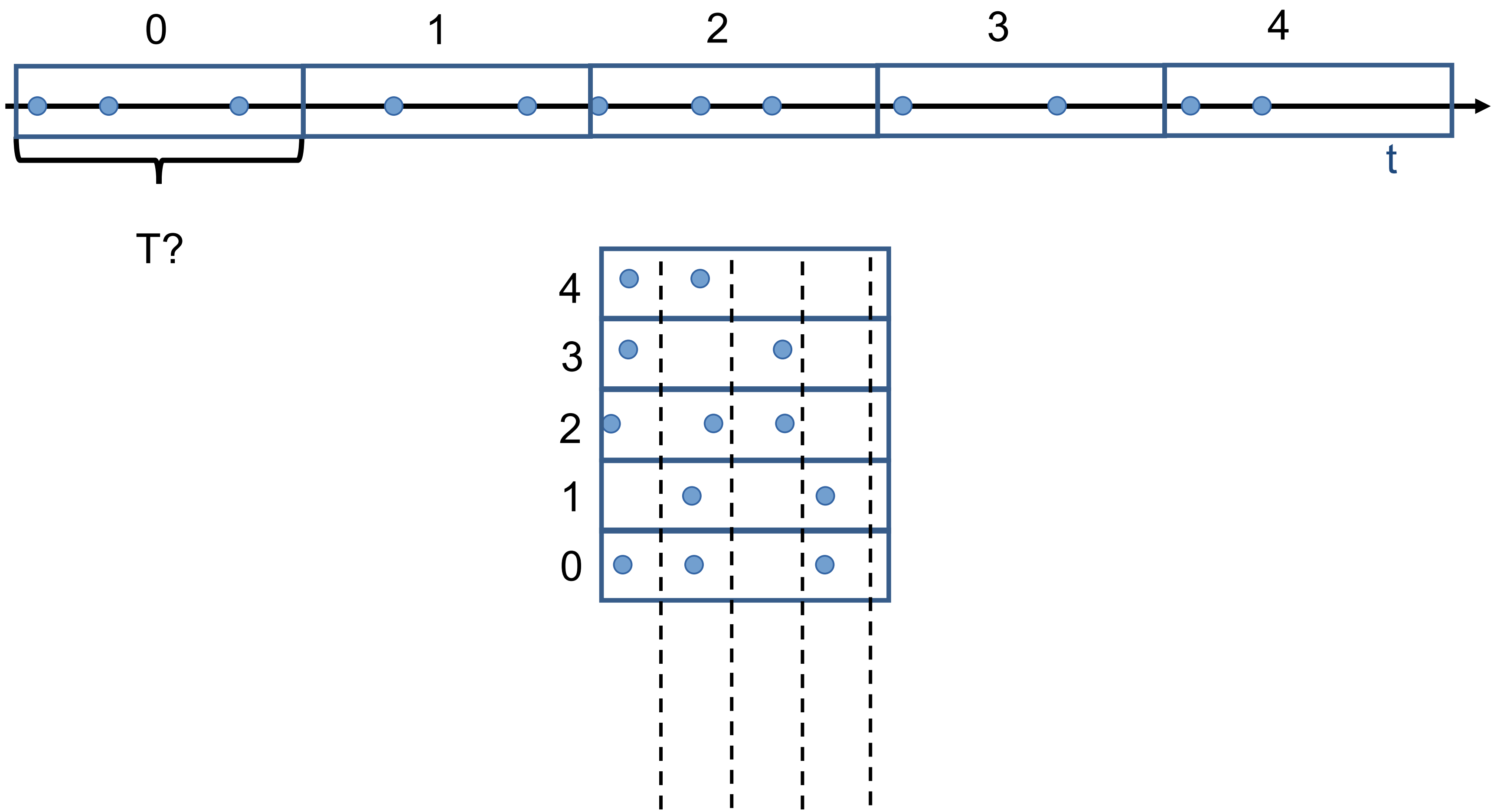

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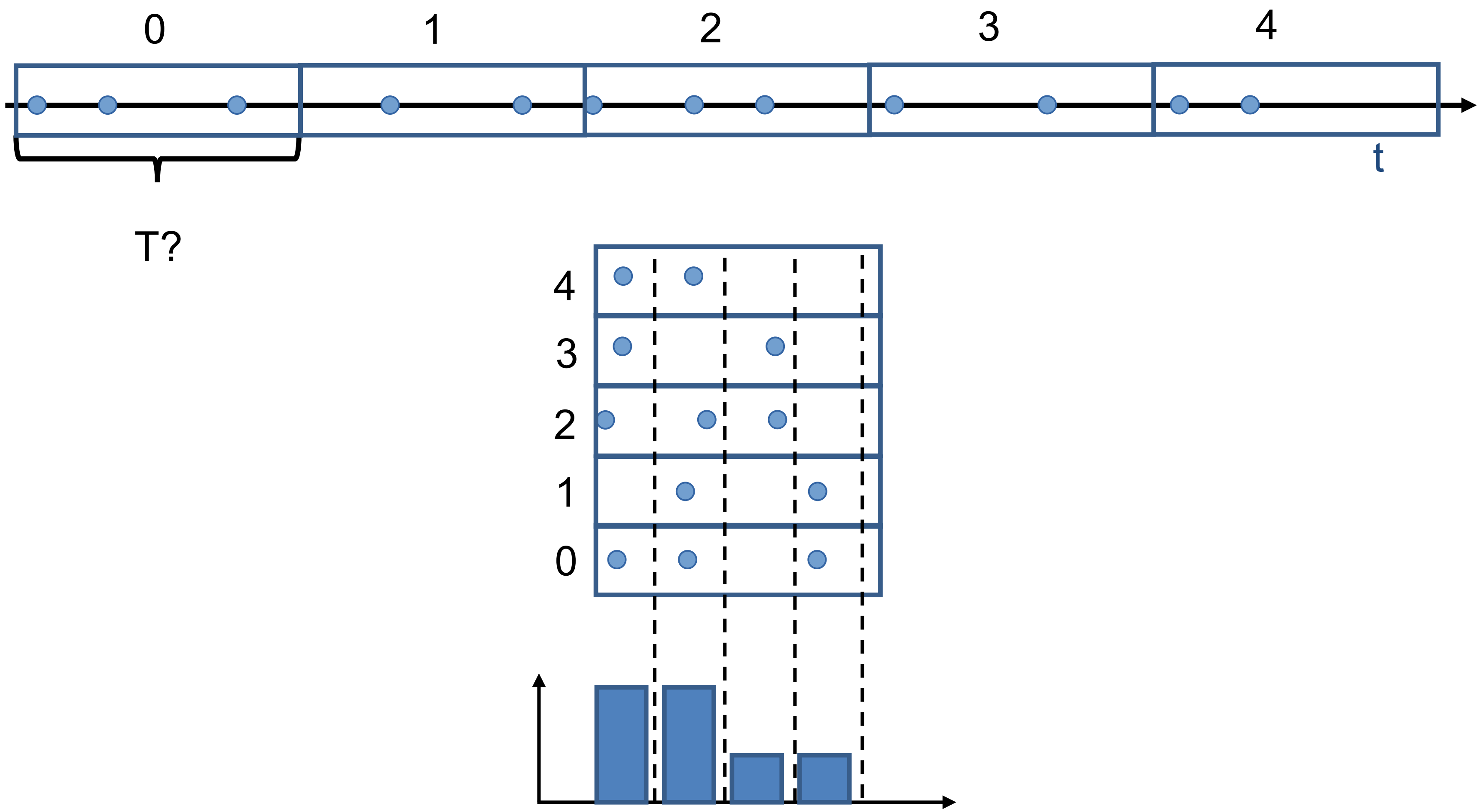

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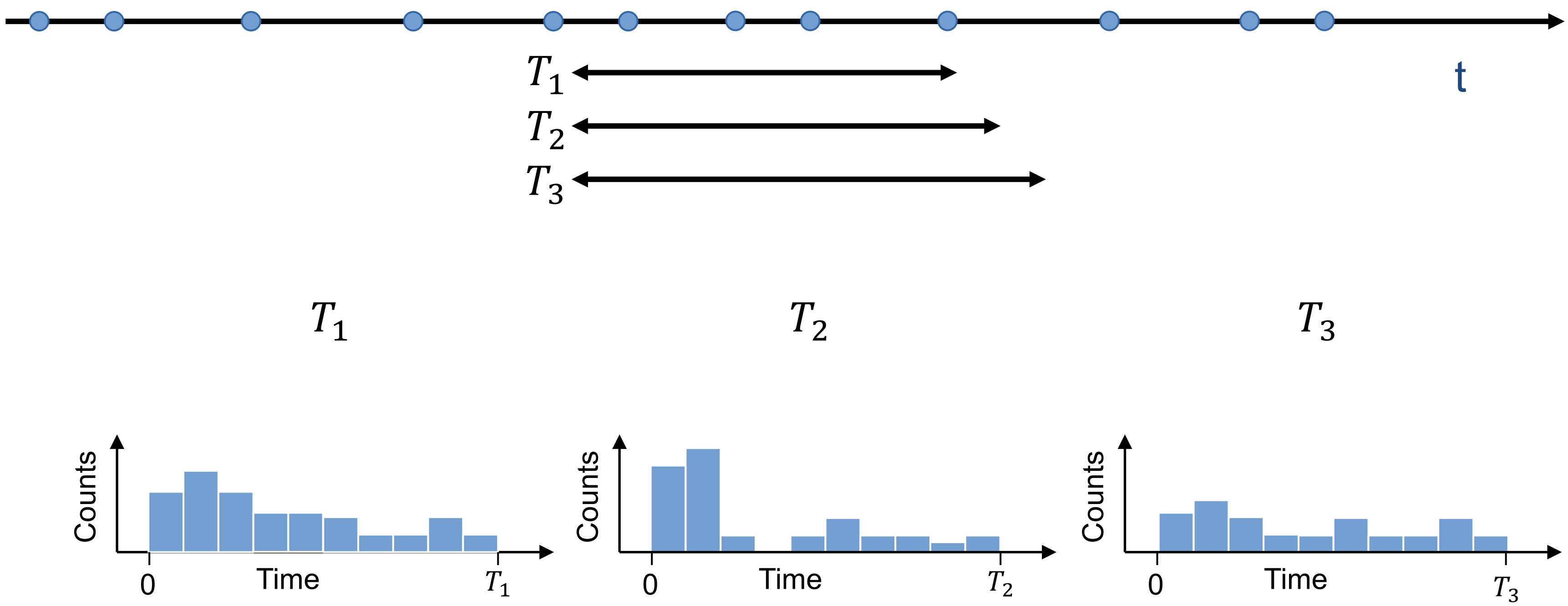

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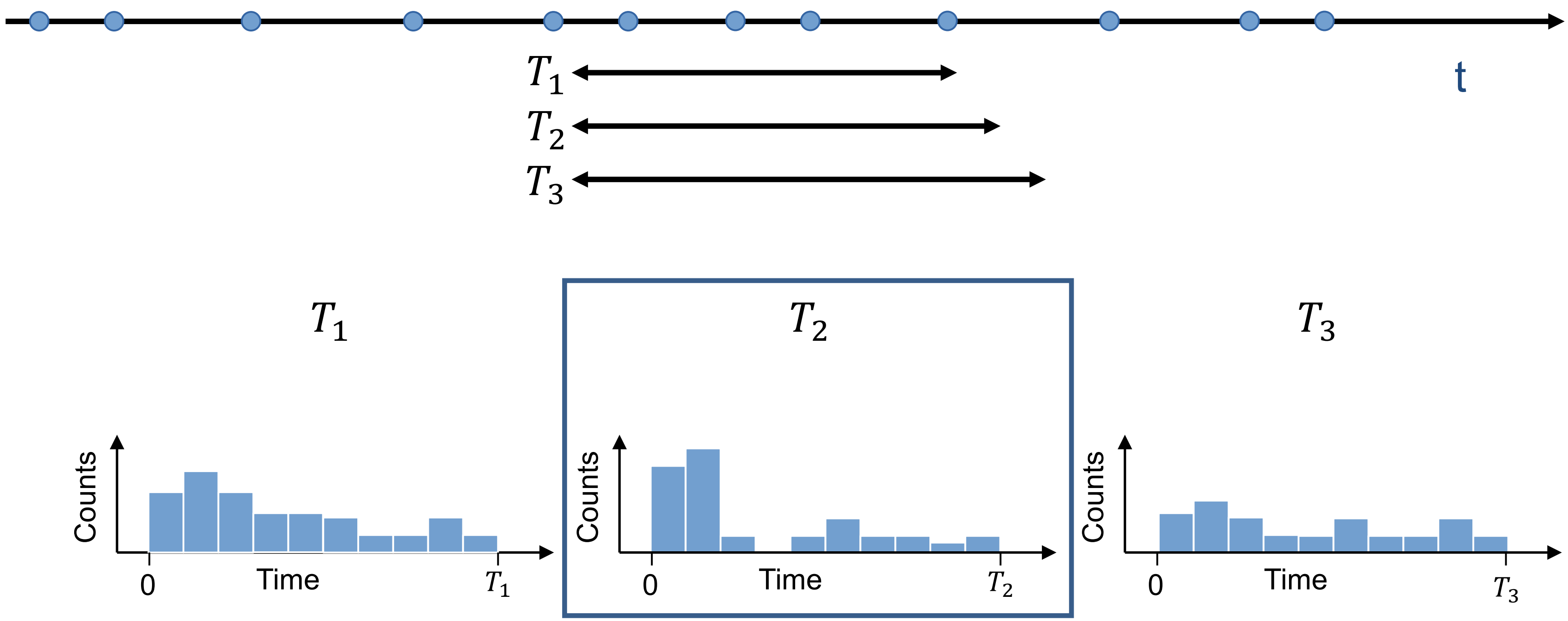

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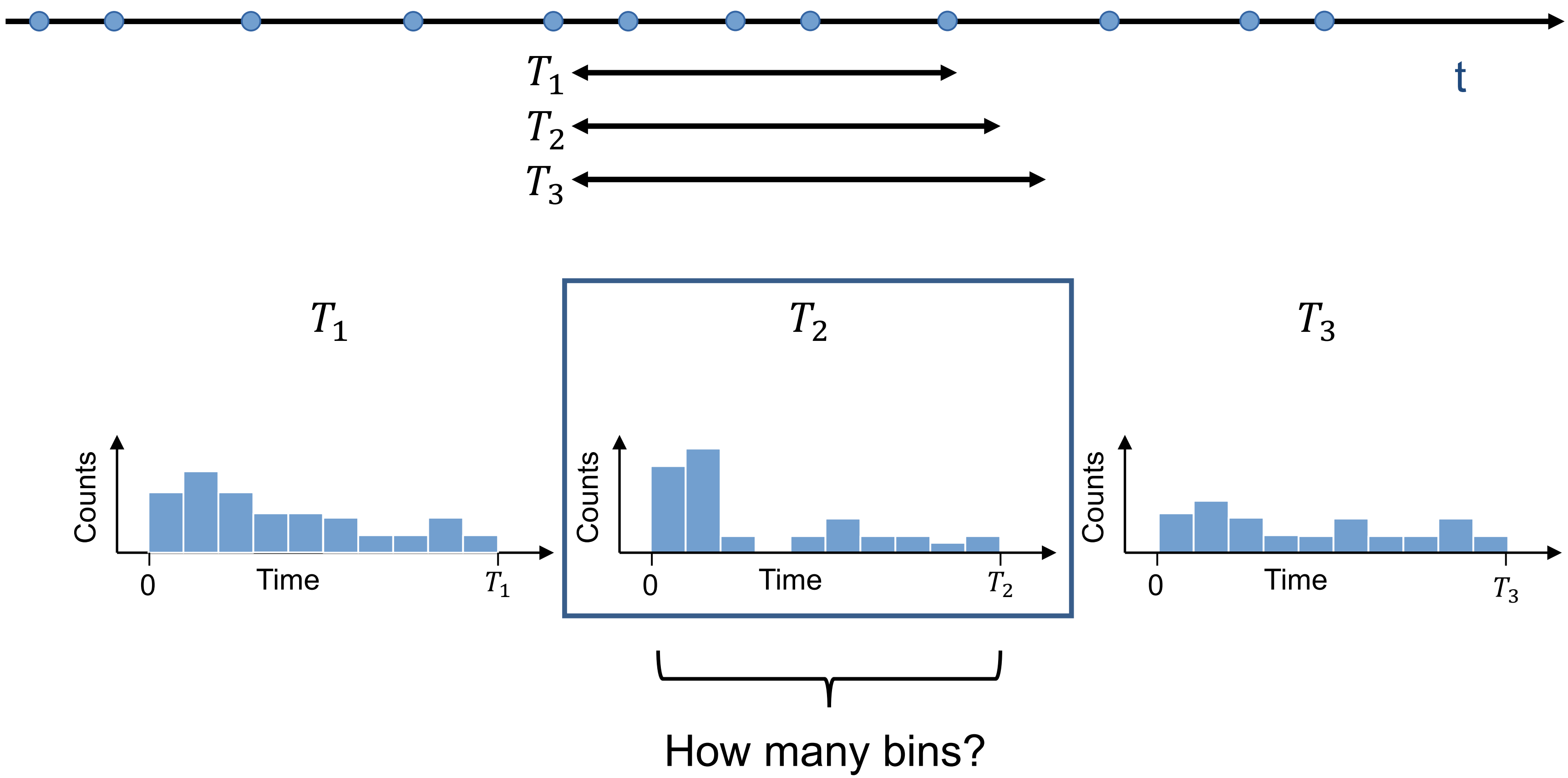
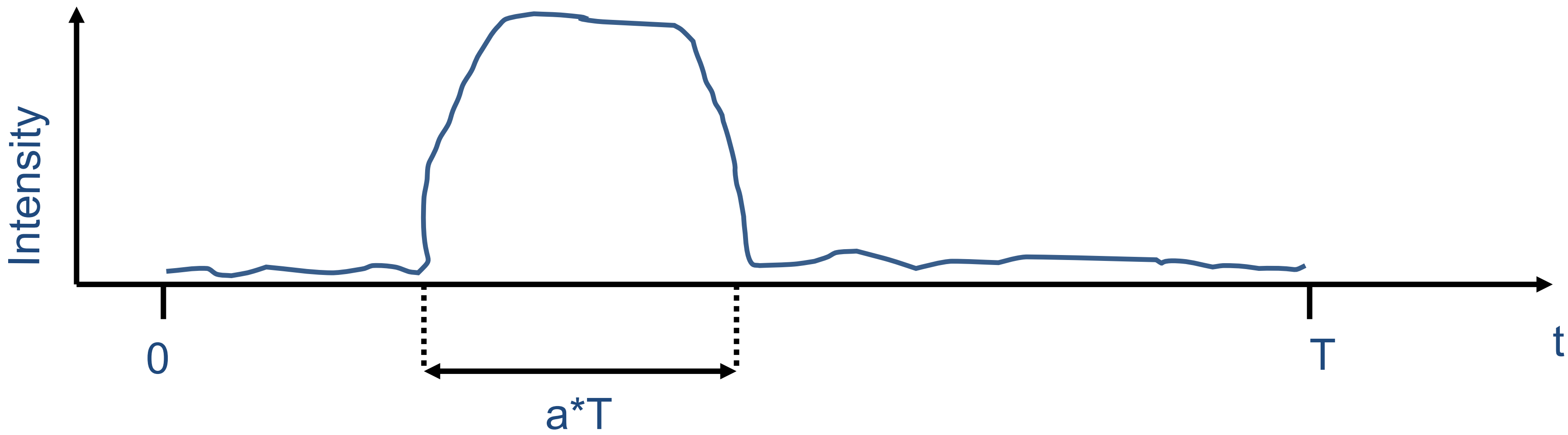


How many bins?

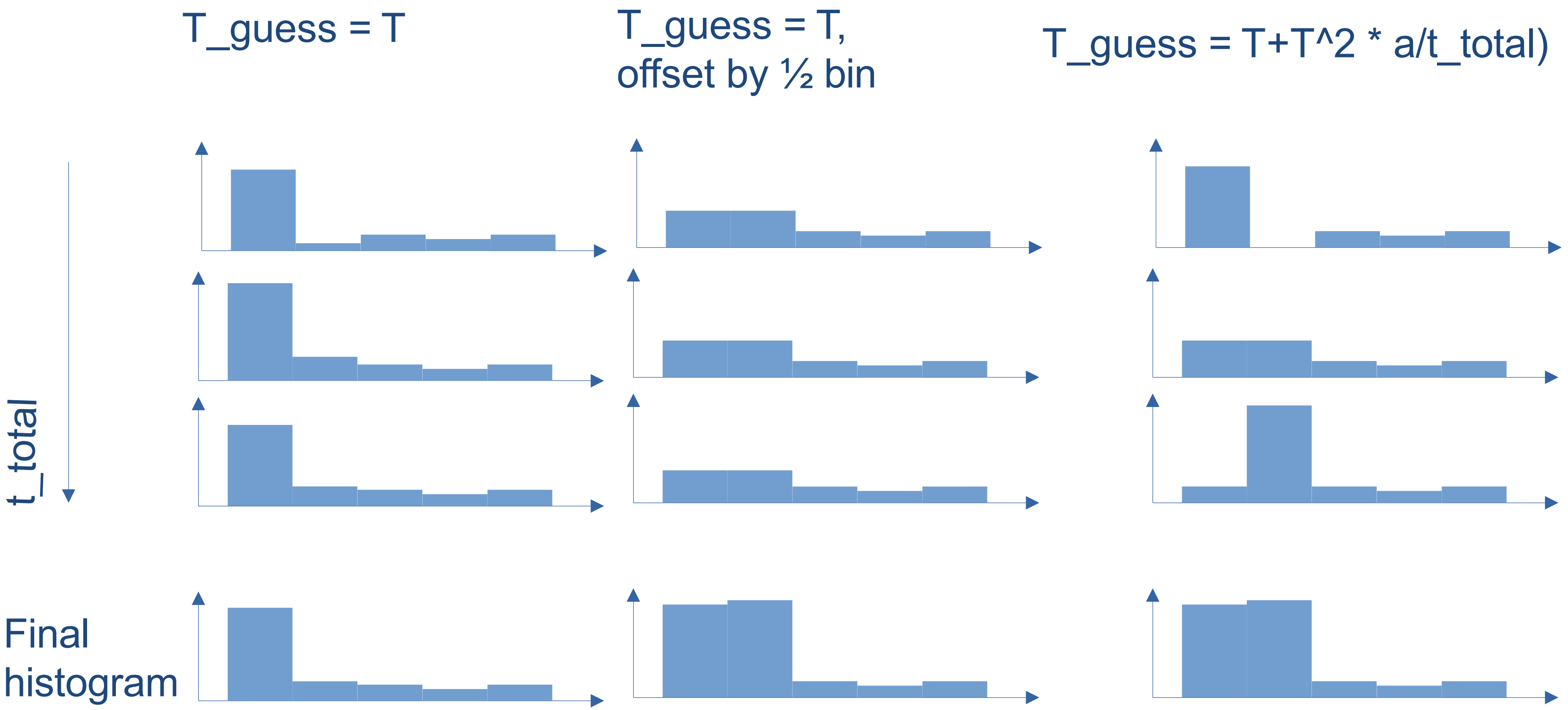
- Guess temporal signal distribution
- → in period T, how long is the signal phase a*T?



- Use 2/a bins

How many bins?

- Why 2/a bins and not 1/a?

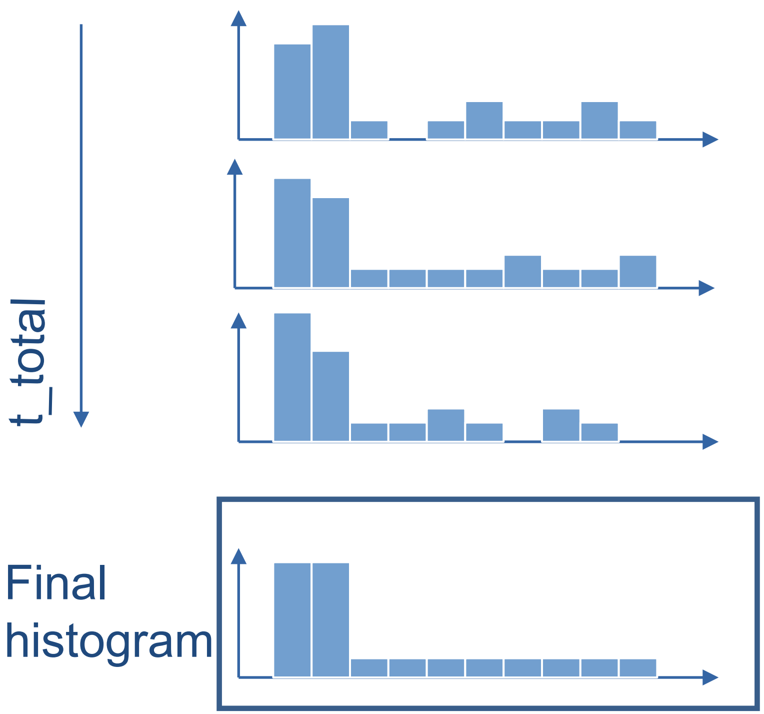


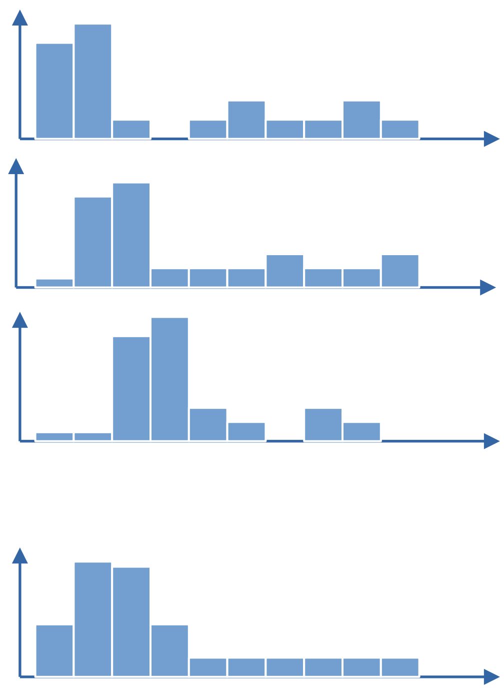
Frequency spacing

- Bandwith of variance?

$$T_{guess} = T$$

$$T_{guess} = T + aT \cdot \frac{T}{t_{total}}$$



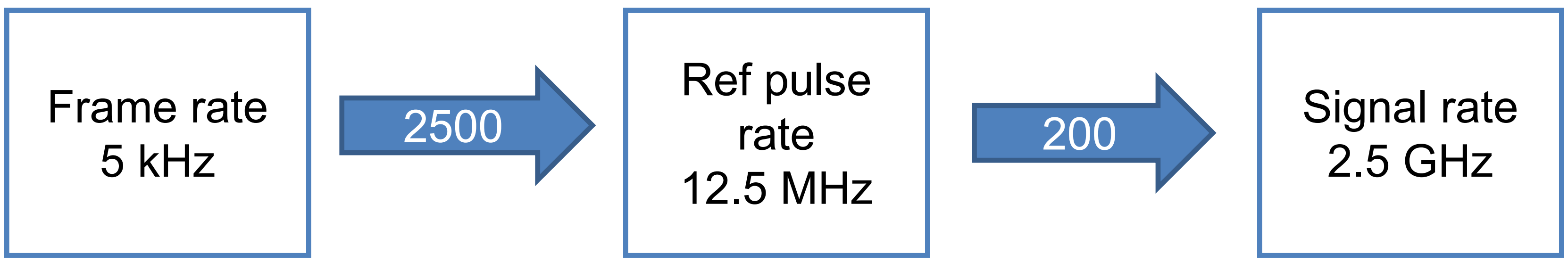


Frequency spacing

- Period array resolution at least $a \cdot \frac{T^2}{t_{total}}$
- → this is also the highest accuracy one can achieve with limited t_{total} (→ “FWHM”)
- Exact ratio between $var(T_{correct})$ and $var(T_{wrong})$ depends on dark and background counts (=noise) n_{dc}, amount of clicks n_S and the temporal shape of the signal.

Finding frequencies

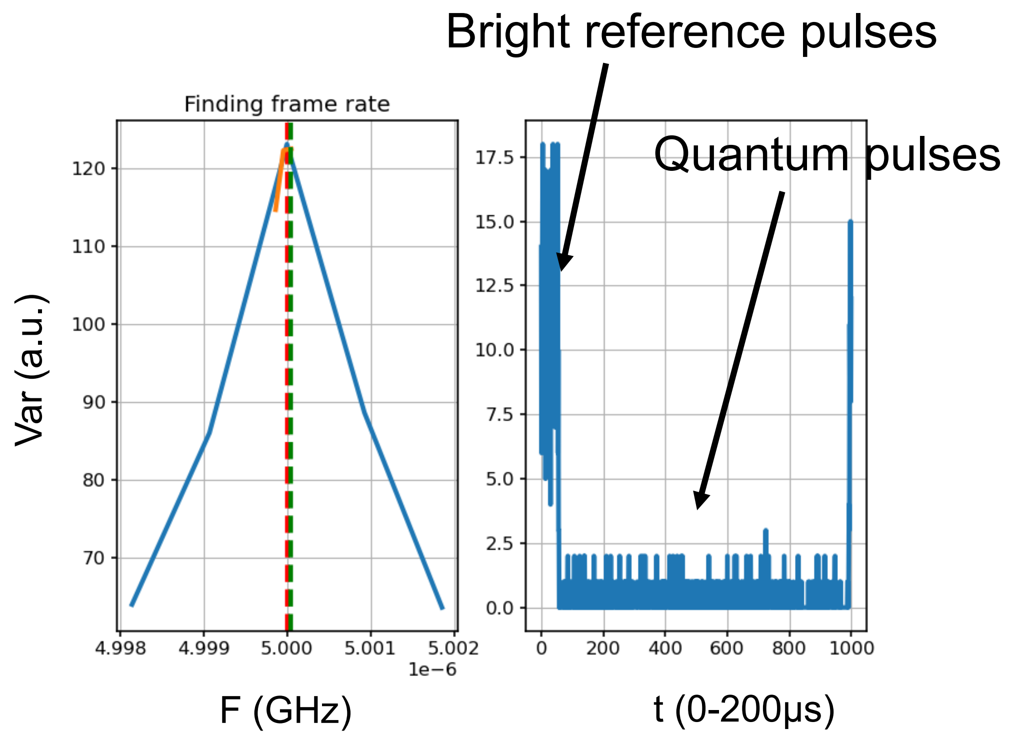
- Start with low frequencies (frame rate), work forward to high frequencies (signal rate)



- Integer factors between them! → allows to use all recorded data without adding artifacts

Finding frequencies

- Start with low frequencies (frame rate), work forward to high frequencies (signal rate)
- Higher frequencies are integral multiples of the lower ones



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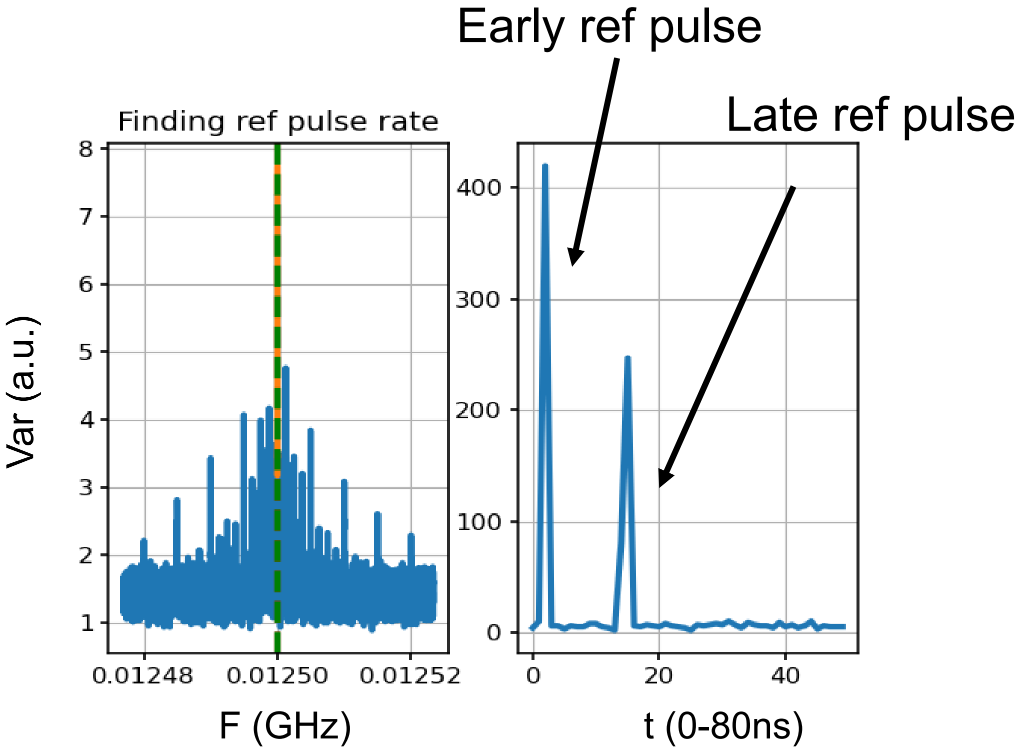

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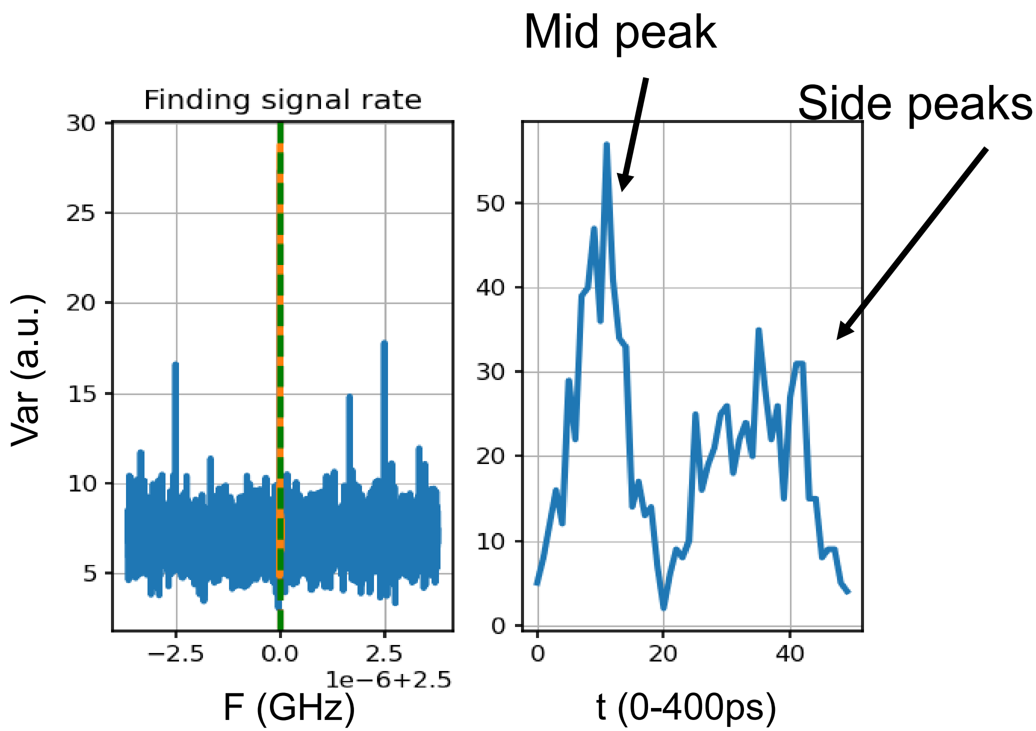
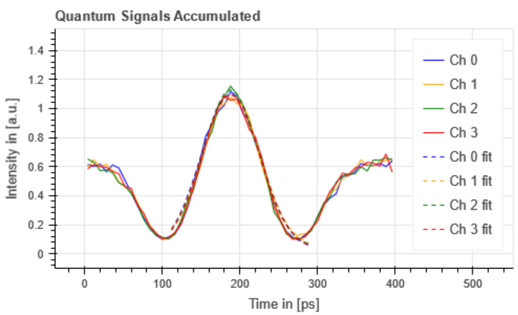
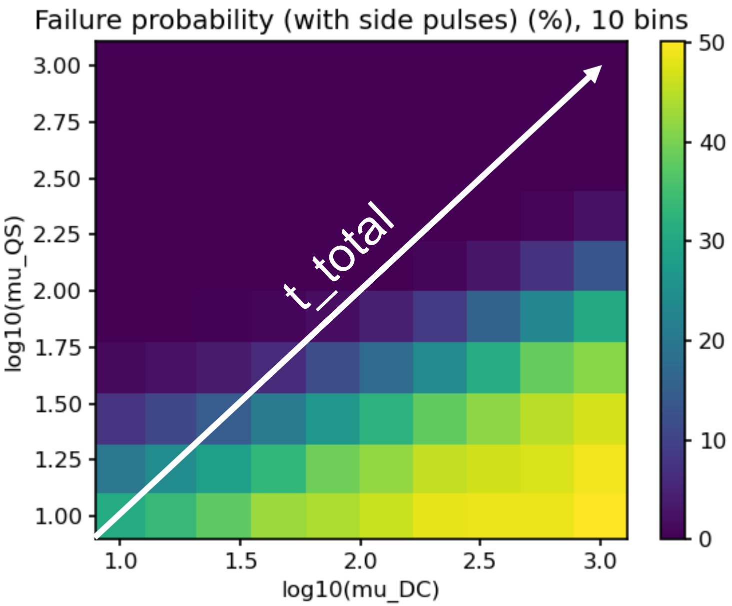


Failure probability

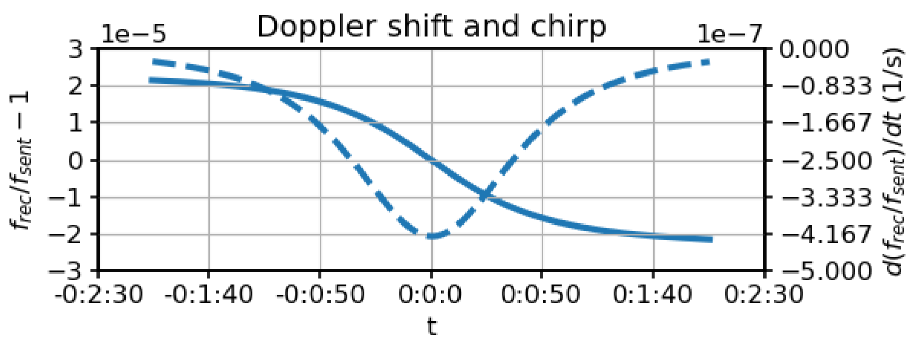
- How does epoch folding perform with low count rates and high noise → QKD?
- → Monte Carlo simulations
- As expected: increasing signal to noise ratio or measurement duration decreases failure probability





Likelihood of failure

- Easy, just increase t_{total} until failure probility=0!



- Chirp γ_1 (LEO orbit!) limits t_{total}

$$t_{total} < \sqrt{\frac{2aT}{\gamma_1}} = 20\ ms$$

- Luckily, strongest chirp during zenith → better SNR

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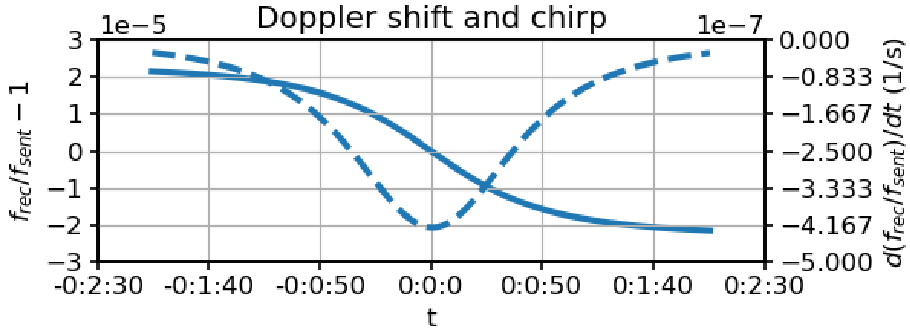

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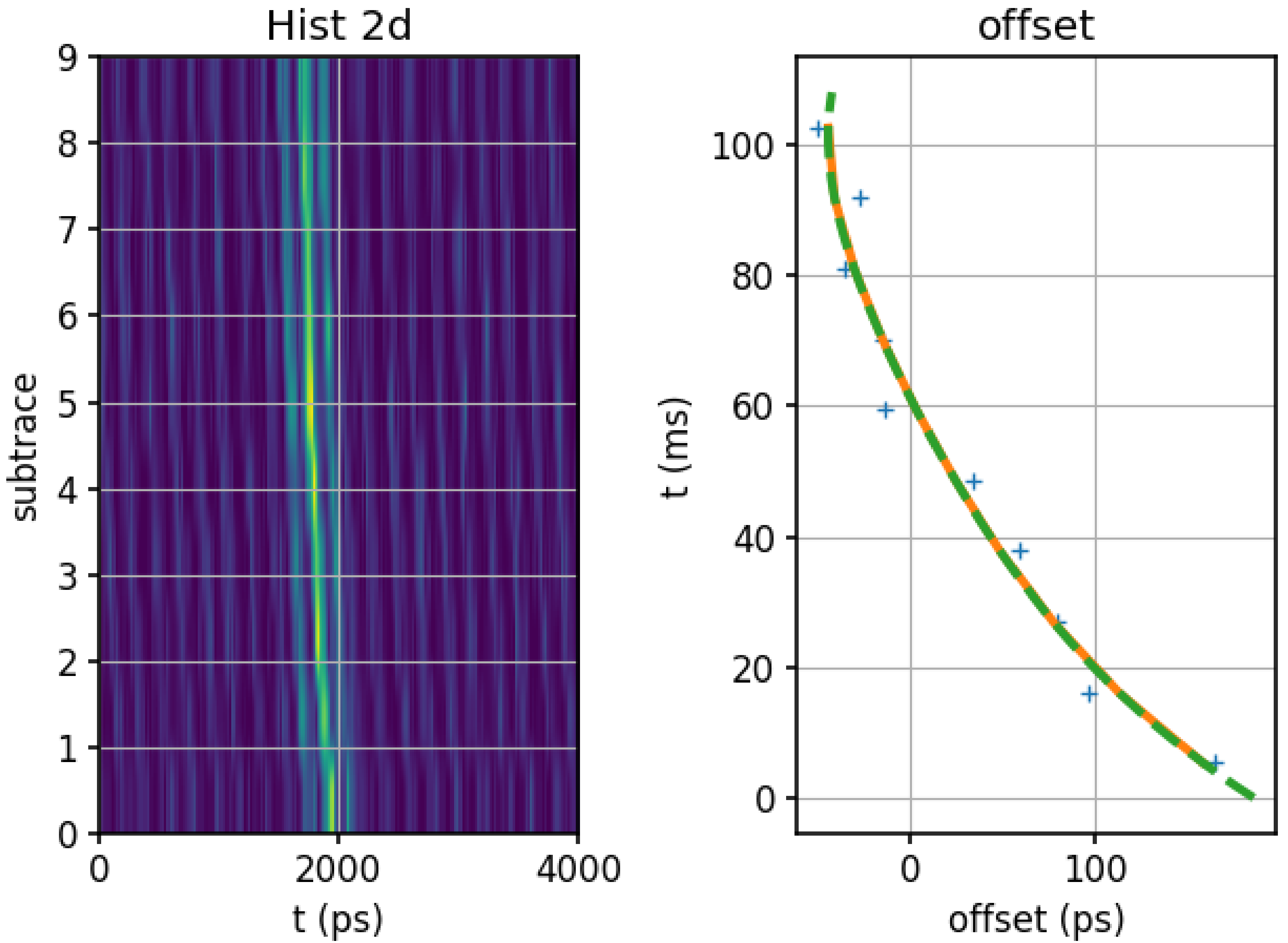
Removing chirp

- Signal rate is time dependent
- Easy approach:
  - divide recorded trace in subtraces
  - Get signal rate f_S per subtrace
  - Cubic fit $1/f_S \approx a_0 + a_1 \cdot t + a_2 \cdot t^2 + a_3 \cdot t^3$

$$t_S = \sum_{k=1}^{3} t_R^k \cdot a_k \Big/ k\,!$$

Removing chirp

- **Faster approach**: get signal rate only once for whole trace
- Then cut trace into subtraces, get histograms with the signal rate calculated before
- Since frequency drifts slowly, Δf is visible in a phase offset for each subtrace


Hist 2d
subtrace
t (ps)
0
1
2
3
4
5
6
7
8
9
0
2000
4000
offset
t (ms)
offset (ps)
0
20
40
60
80
100
0
100

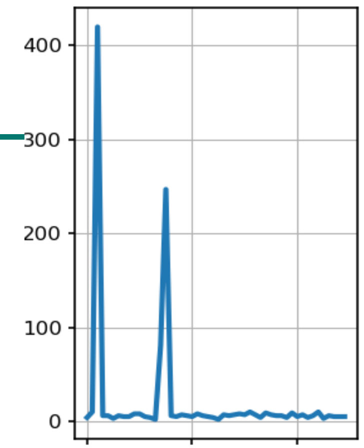
Removing chirp

- Now we can find a function for the temporal offset $\Phi(t)$
- The doppler shift $\gamma(t)$ is then defined as
- $\gamma(t) = \frac{d\Phi(t)}{dt}$

$$t_S = \sum_{k=1}^{3} t_R^k \cdot a_k \Big/ k\,!$$

- Now we can remove the chirp and find a single signal rate for the complete recorded trace
- **Relative synchronization ✓**

Frame number decoding



- **Relative synchronization ✓**
- → Absolute sync?
- → Frame numbers, encoded in the reference pulses via early/late
  - 5k frames per second
  - 158 reference pulses within frame every 80 ns
  - First 127 pulses: PRBS sequence
  - Last 31 pulses: frame number

t

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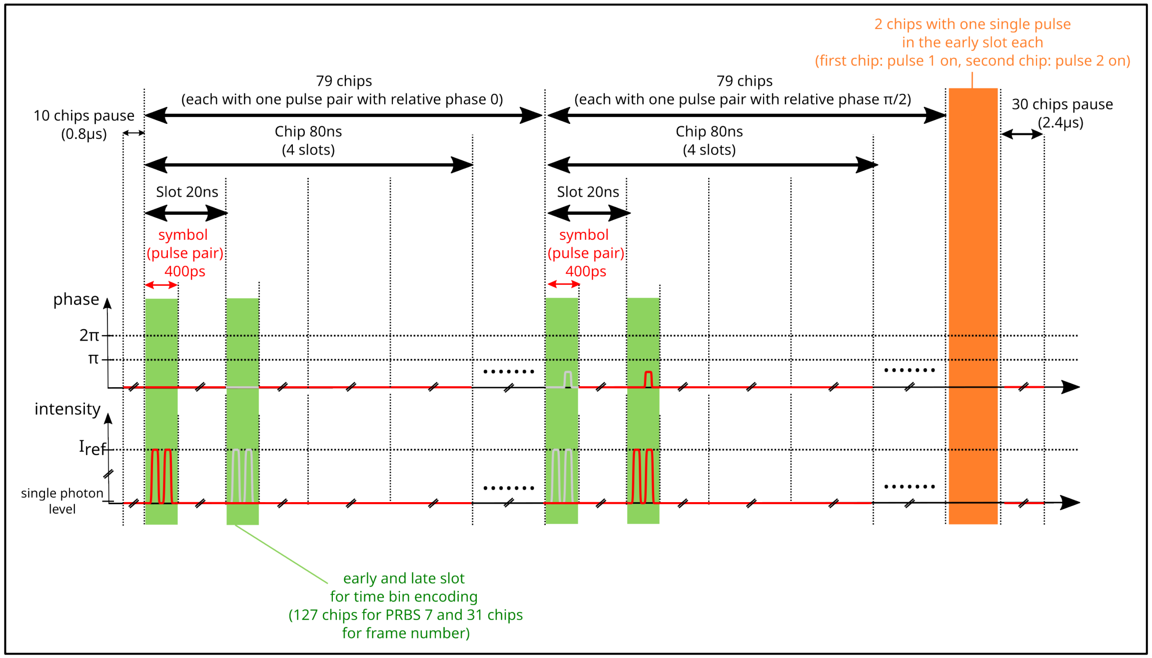
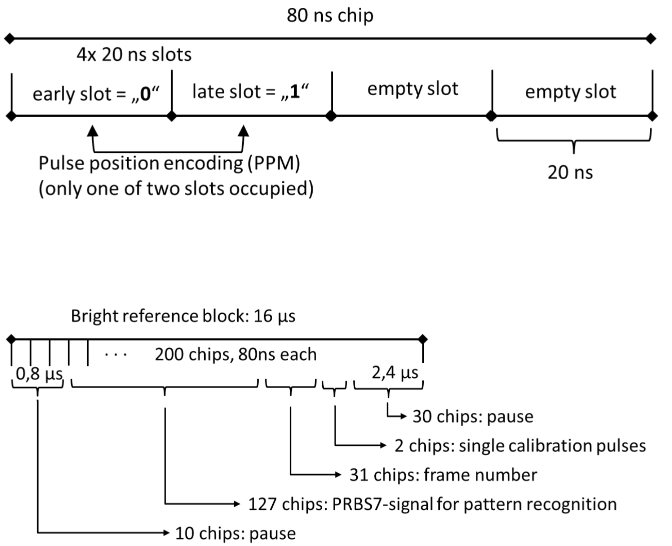
Frame number decoding

QKD Protocol definition: Bright Reference Pulses



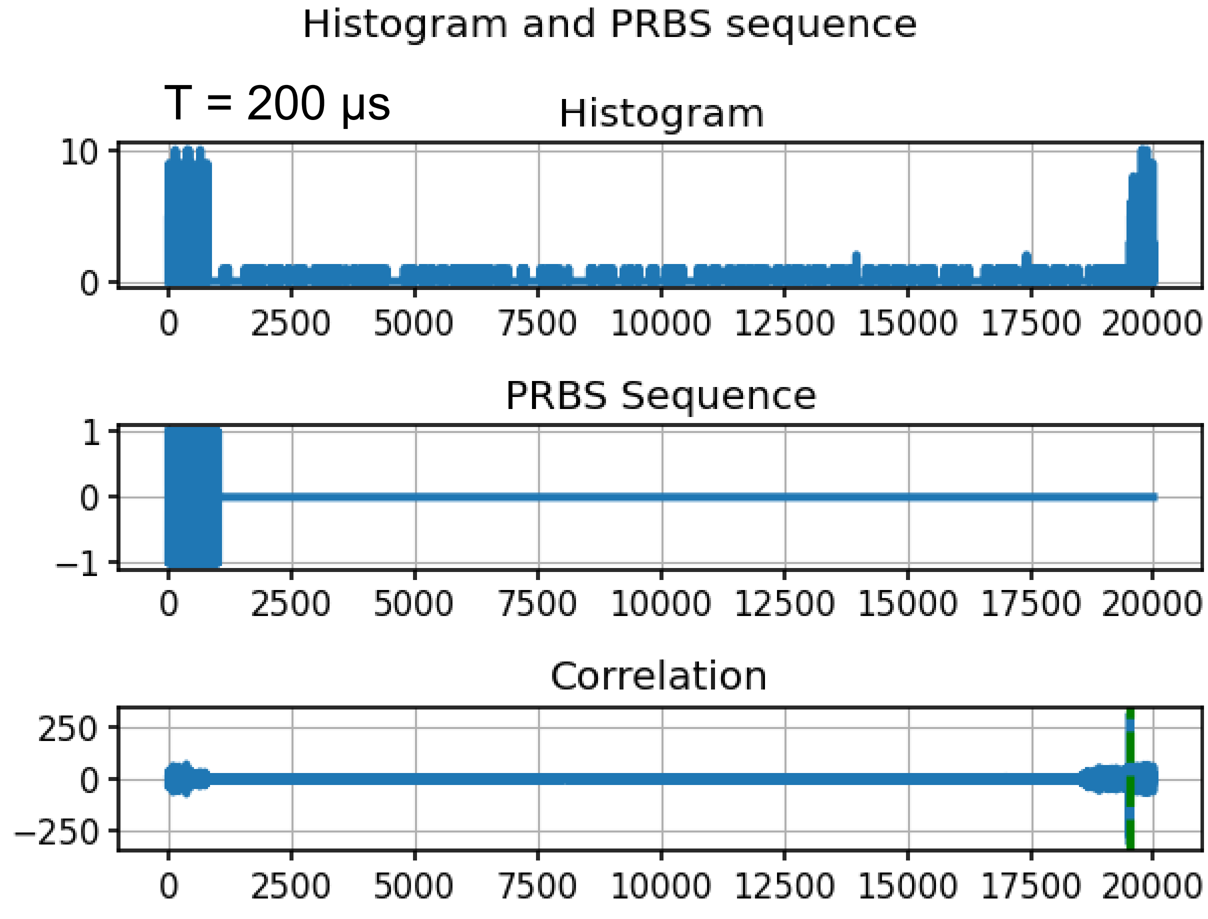
2. Frame Number PPM encoding:



→ Bit synchronization with satellite without absolute time synchronization

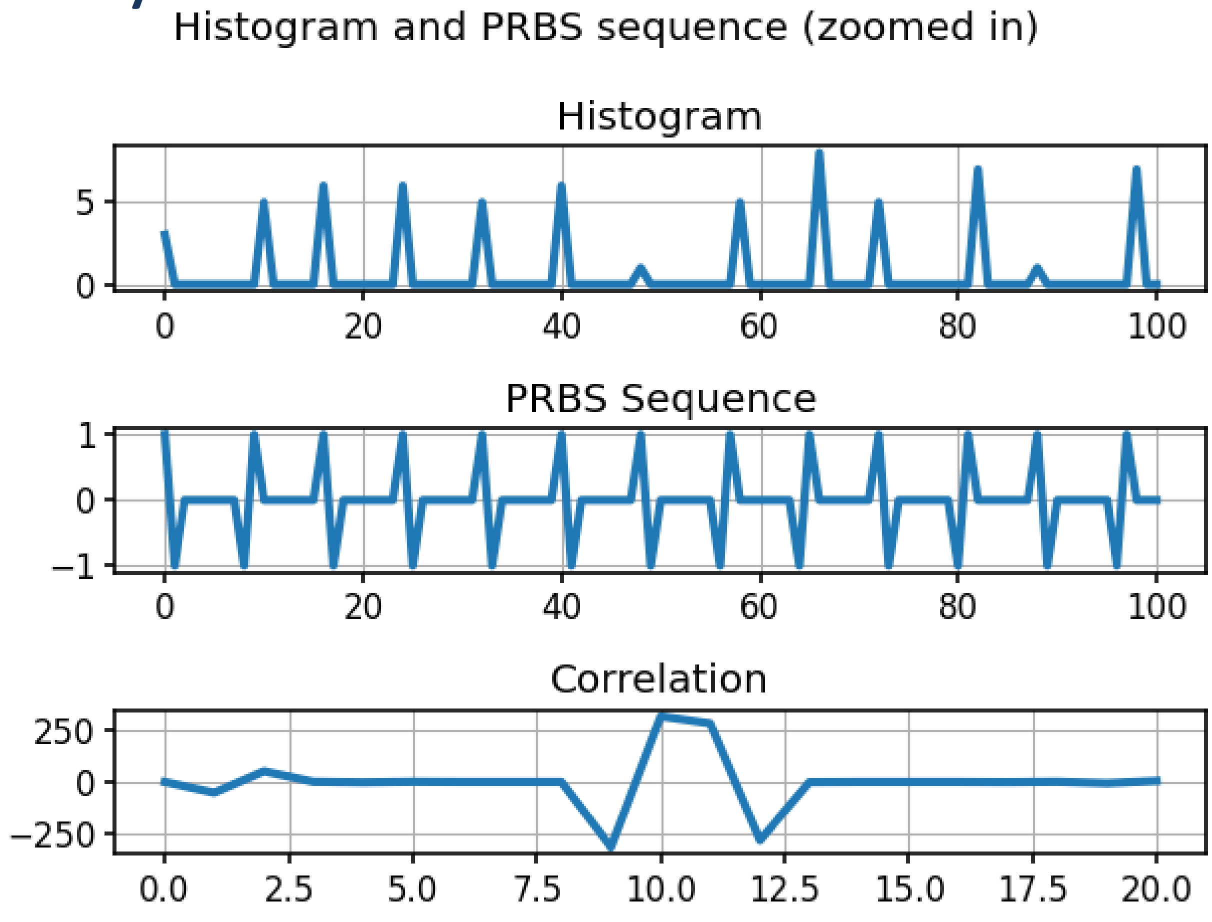
Find offsets!

- Find start of frame via PRBS correlation



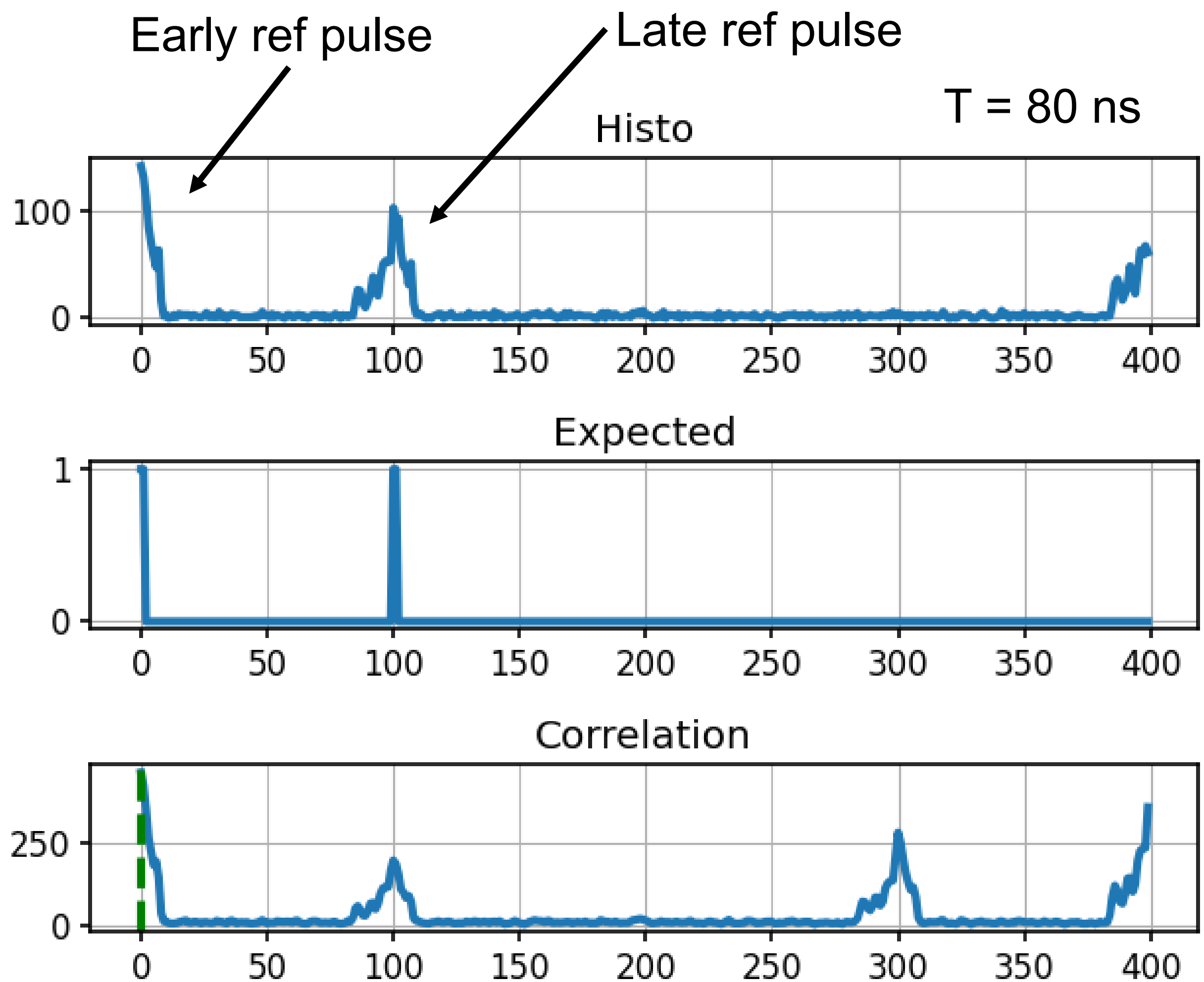
Find offsets!

- Find start of frame via PRBS correlation
- → ~10 ns accuracy



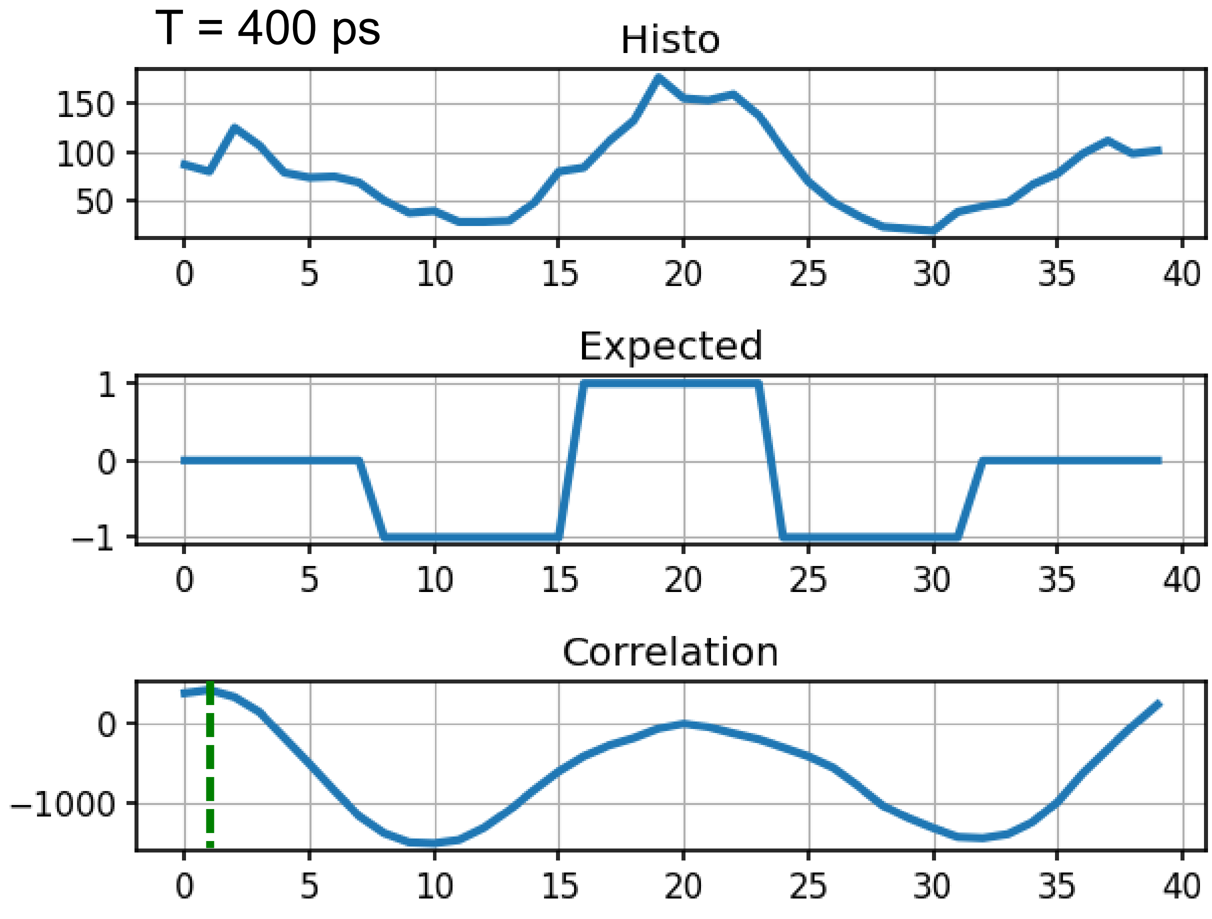
Find offsets!

- Increase accuracy with higher frequencies
- → ~ 1 ns



Find offsets!

- Increase accuracy with higher frequencies
- → ps accuracy



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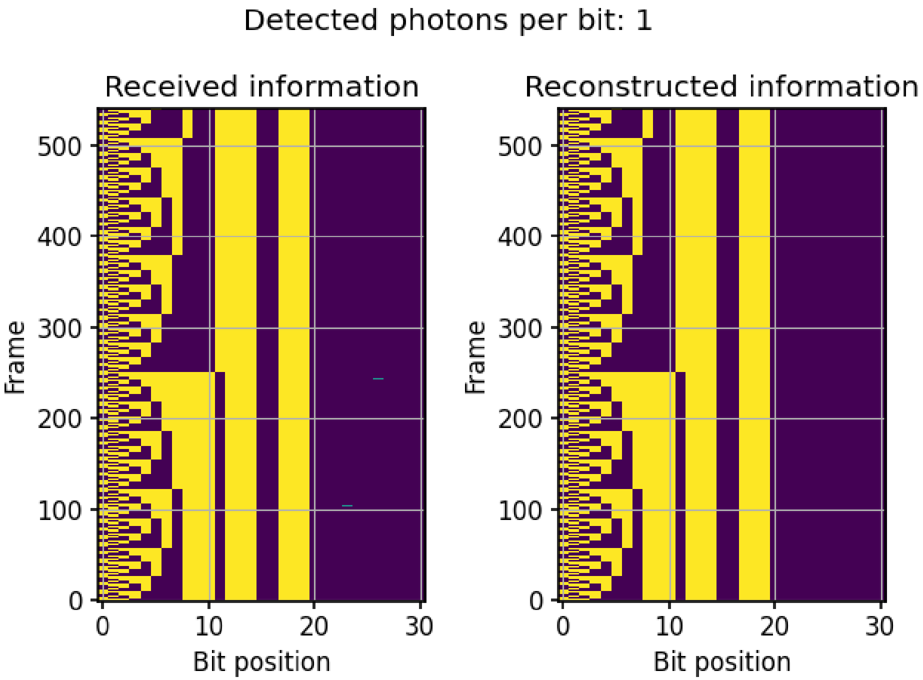
Frame number decoding

- Find offset with PRBS correlation
- Decode information using monotonously increasing bits



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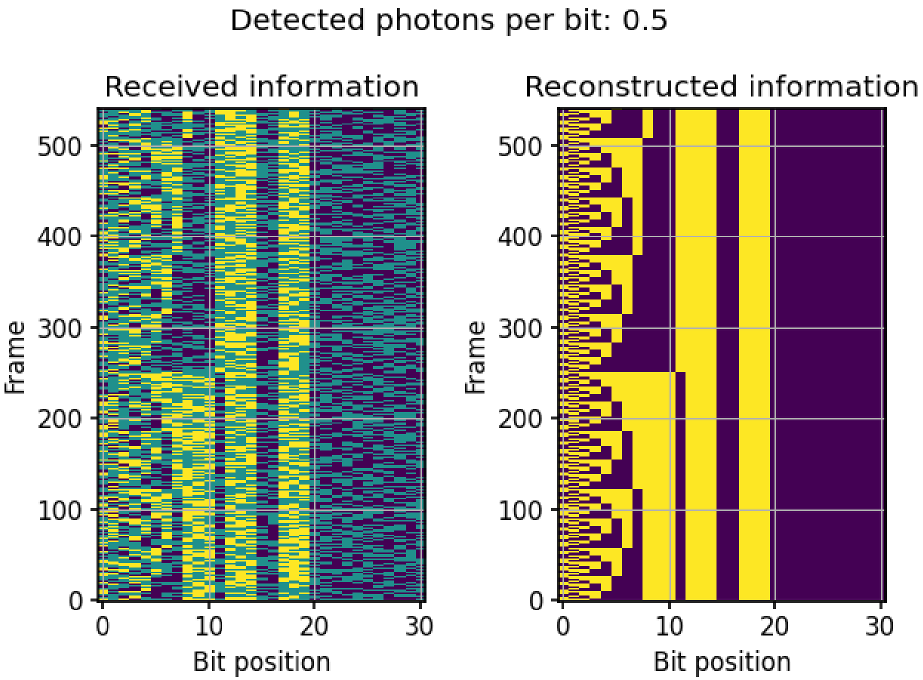

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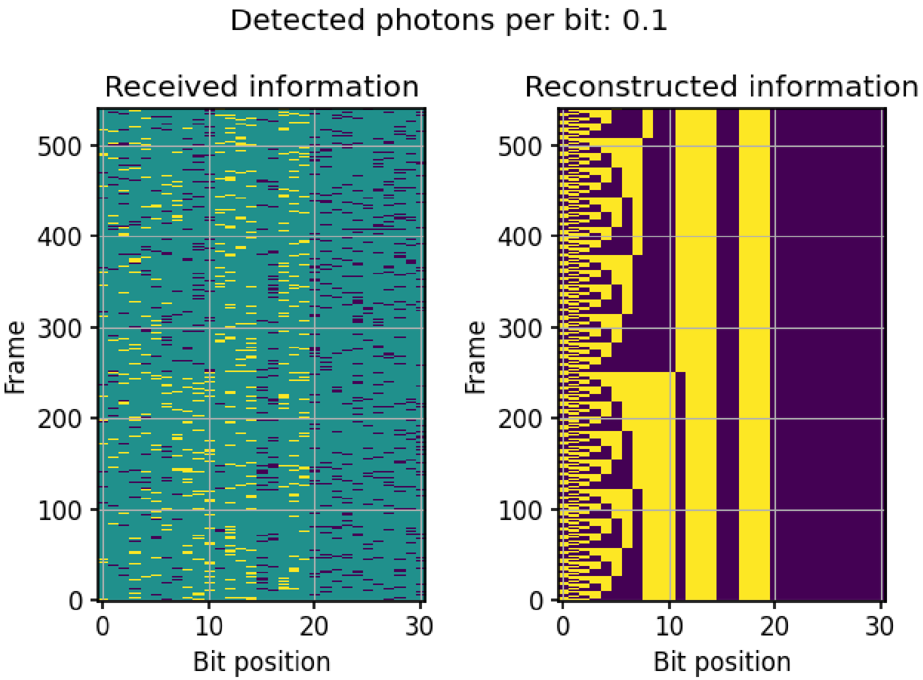

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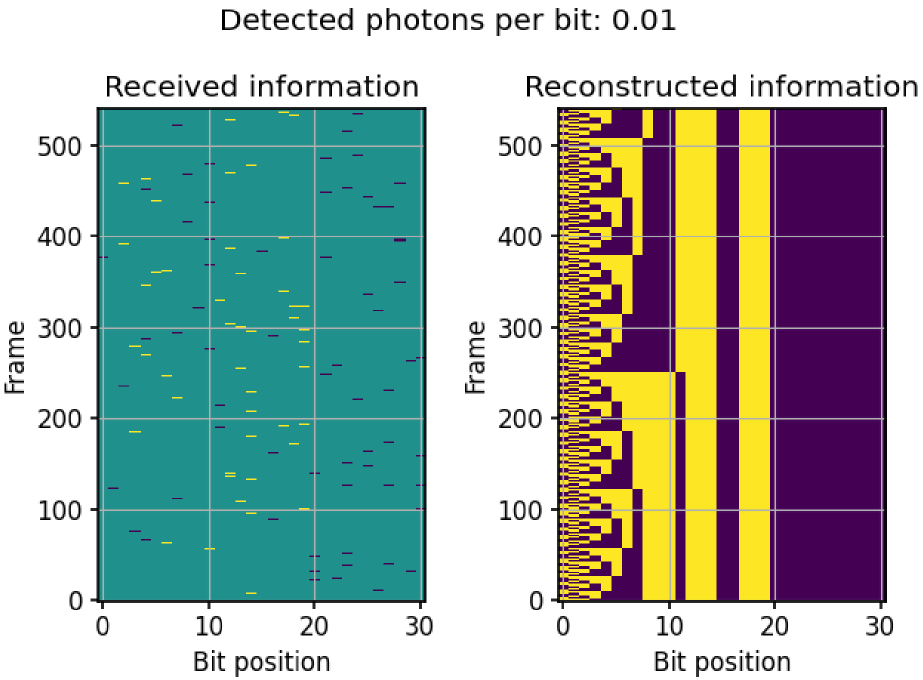

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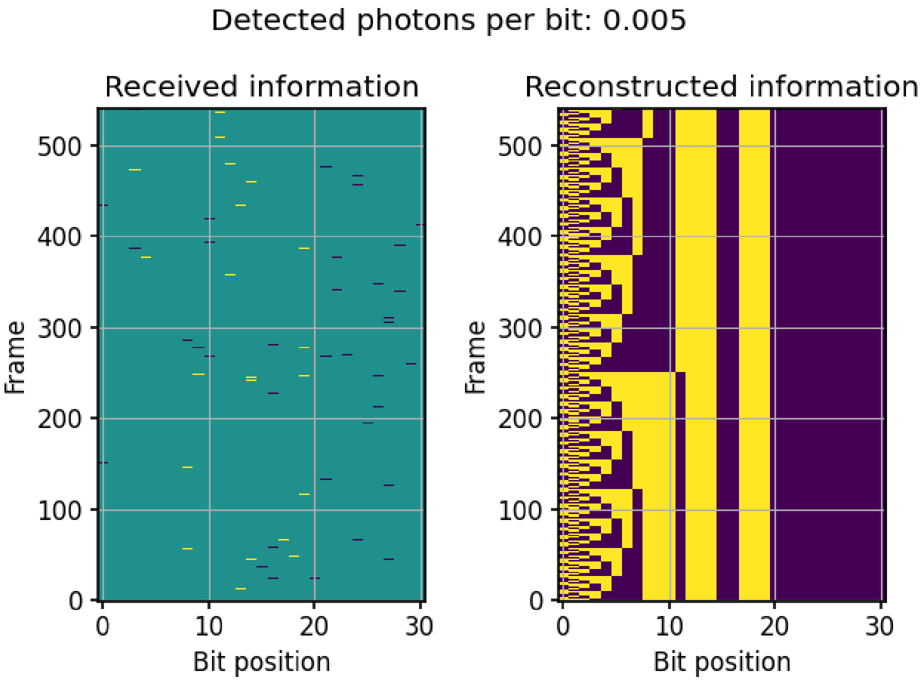
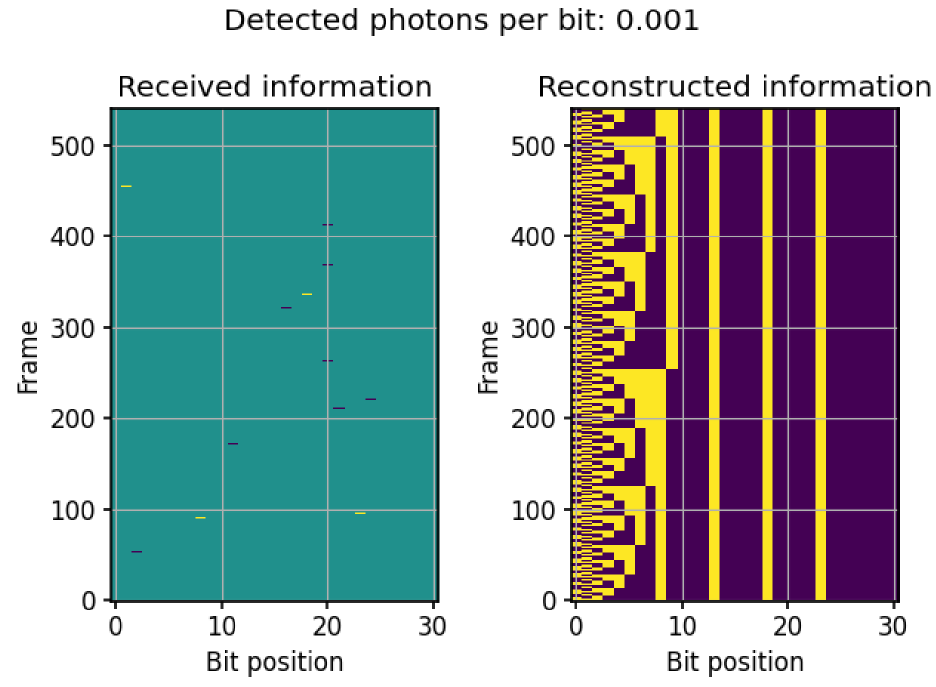


Frame number decoding

- Find offset with PRBS correlation
- Decode information using monotonously increasing bits
- Very robust, works with 0.01 received photons per bit



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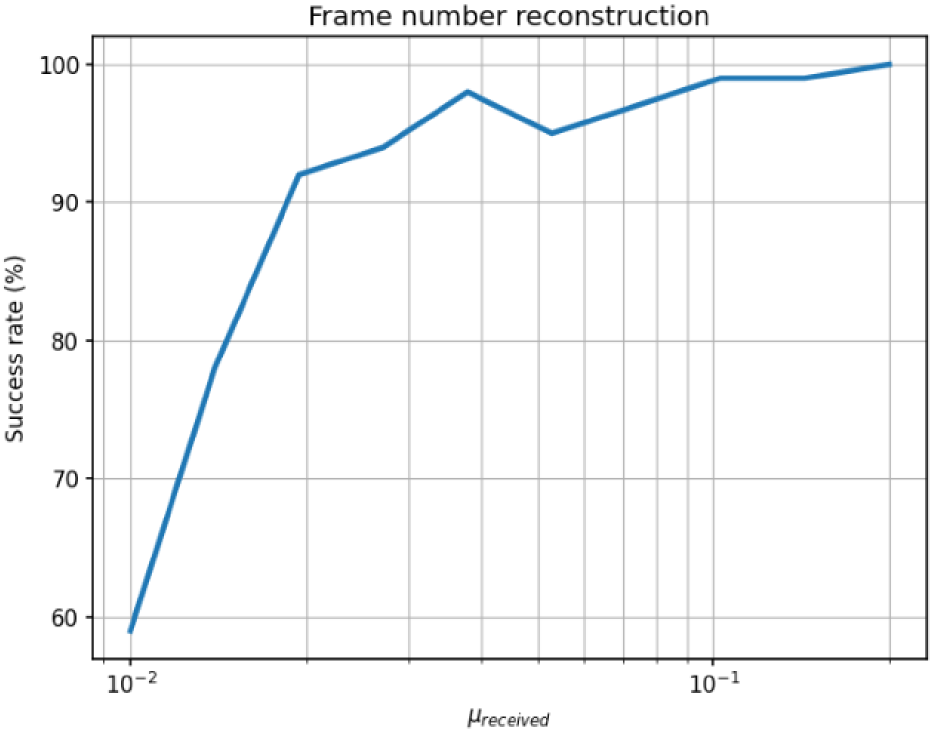
Frame number decoding

- Find offset with PRBS correlation
- Decode information using monotonously increasing bits
- Very robust, works with 0.01 received photons per bit
- **Absolute sync ✓**



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Summary

[Scientific publication in preparation]

- **Relative sync ✓**
  - Epoch folding
  - Higher frequencies are integral multiples of lower ones
- **Absolute sync ✓**
  - Correlations and frame numbers

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Friedrich-Alexander-Universität Erlangen-Nürnberg

Thank you for your attention!

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